Efficiency of Health Systems in Sub-Sahara Africa: A Comparative Analysis of Time Varying Stochastic Frontier Models

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ABSTRACT---- ***The purpose of the current study was to estimate efficiency of health systems in sub-Sahara Africa (SSA) and to compare efficiency estimates from various time-varying frontier models. The study used data for 45 countries in SSA from 2005 to 2011 sourced from the Word Bank World Development Indicators. Parametric time-varying stochastic frontier models were used in the analysis. Infant survival rate was used as the outcome variable, while per-capita health expenditure was used as main controllable input. The results show some variations in efficiency estimates among the various models. Estimates from the 'true' random effect model were however preferable after controlling for unobserved heterogeneity which was captured in the inefficiency terms of the other frontier models. The results also suggest a wide variation in the efficiency of health systems in sub-Sahara Africa. On average health system efficiency was estimated to be approximately 0.80 which implies resource wastage of about 0.20. Cape Verde, Mauritius and Tanzania were estimated to be relatively efficient while Angola, Equatorial Guinea and Sierra Leone were among the least performers in terms of health system efficiency. The findings suggest that the omission of unobserved heterogeneity may lead to bias in estimated inefficiency. The 'true' random effect model was identified to address the problem of unobserved heterogeneity. The findings also suggest a generally poor performance of health systems in terms of efficiency in the use of resources. While resource commitment to the health sector is critical, it is important to also ensure the efficient use of these resources. Improving the performance of institutions in the health sector may go a long way in improving the general health status of the African population.***

Keywords--- Efficiency, Health systems, health expenditure, SSA, SFA, 'True' random effect

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1. INTRODUCITON

Sub-Sahara Africa (SSA) continues to face major health challenges including high disease burden and poor health care system infrastructure. For instance, life expectancy at birth in SSA was reported to be 55 years in 2011, relative to the world average of 70 years. This also significantly falls short of values for all other regions of the world. Maternal, infant and under five mortality remain high in the SSA region, relative to other regions of the world and the world average (World Bank, 2012).

It has also been reported that the Africa region lags behind in achieving the health-related MDG targets with most countries in the region unlikely to achieve these targets. HIV/AIDS, malaria and tuberculosis remain the major causes of mortality and morbidity in the region with estimated incidence of 217; 21,537 and 276; per 100,000 population, respectively in 2009 (WHO, 2012). The World Health Organization (WHO) in 2011 also showed that only eight countries were on track to achieve the health related MDGs. Majority of the countries in the region were achieving less than 50% of what is expected to reach the target in 2015, with progress on MDG 5 (maternal mortality) being particularly slow.

The ramifications of these poor health performances on household welfare, productivity and economic growth cannot be over emphasised. The SSA region is estimated to have the smallest GDP per capita, relative to all other regions of the world. GDP per capita in purchasing power parity terms was about US$2362.90 in 2011 which was an increase from about US$1389.70 in 2000. The region also remains one of the poorest regions in the world with high rates of poverty and relatively more impoverished households. For instance the percentage of population in SSA living below US$1.25 and US$2.00 a day was estimated to be 48.5% and 69.9% in 2010, respectively, higher than any other region of the world (World Bank, 2010).

The health system is widely considered as an important institution in the health improvement agenda of any country (Hakkinen and Joumard, 2007). The world health report of 2000 considers health systems to be crucial in the development of individuals, families and societies everywhere with three intrinsic goals: improving health, increasing responsiveness to the legitimate demands of the population and ensuring that financial burdens are distributed fairly (WHO, 2000). Achieving these goals seems to be slow in SSA where population health remains poor and financial burdens largely on individuals. Improving this situation requires a comprehensive analysis of health systems in the region including the relationship between health system inputs and outputs as well as efficiency of the health systems in the use of resources (Powell-Jackson *et al.*, 2012). For instance the WHO (2012) noted that high or low levels of health funding might not translate into improved health outcomes but rather efficiency and equity in the use of these resources. This argument is truly justified in the case of SSA considering the enormous burden of diseases and other health challenges in the region.

Achieving the objective of a well performing health system through improved efficiency also requires accurate and comprehensive measure of efficiency. Evidence from economic literature suggests that the measurement of efficiency has evolved over time with different researchers having different approaches. Danquah et al. (2013) noted that improvements in technical efficiency results in great gains in productivity and economic growth. This implies that, measuring efficiency in an accurate way is critical not only for economic theory but also in providing useful policy information. In this regard, frontier models and particularly the stochastic frontier models have been widely used in the applied economics literature. The basic tenets of the stochastic frontier approach (SFA) lies in the estimation of a stochastic relationship between a set of inputs and outputs of decision making units (DMU). Greene (2004) provides three distinctive properties of the SFA that makes it an attractive alternative to the commonly used data envelopment analysis (DEA) approach. First, the stochastic aspect of the SFA allows it to handle appropriately measurement problems and other stochastic influences that would otherwise be captured as inefficiency. Secondly, the SFA is capable of accommodating unmeasured but substantial cross country heterogeneity. This is particularly important when cross country data is used in analysis. Finally, the SFA also provides a means of employing information on measured heterogeneity in the model.

Further, unlike the cross-section data, the panel data specification of the SFA (which is the focus of the current study) also provides the flexibility of observing DMUs at several points over time hence making a more informative policy decisions. However, a critical concern in the panel data specifications is whether the observations made on the inefficiency term is assumed to be independent over time and across cross-section observations. In this case the panel nature of the data is irrelevant and cross section frontier models can be applied. However, when further assumptions are made about the inefficiency term then, time varying models are used and several possibilities of these models arise. The purpose of the current study is to provide a comparative analysis of the various time varying frontier models with application to estimating health system efficiency in SSA. The rest of the paper is structured as follows. Section 2 provides brief review of literature with focus on the concept of efficiency and some empirical evidence. Section 3 presents the methodology including data and variables. Section 4 presents and discusses the results while section 5 concludes the paper.

2. LITERATURE REVIEW

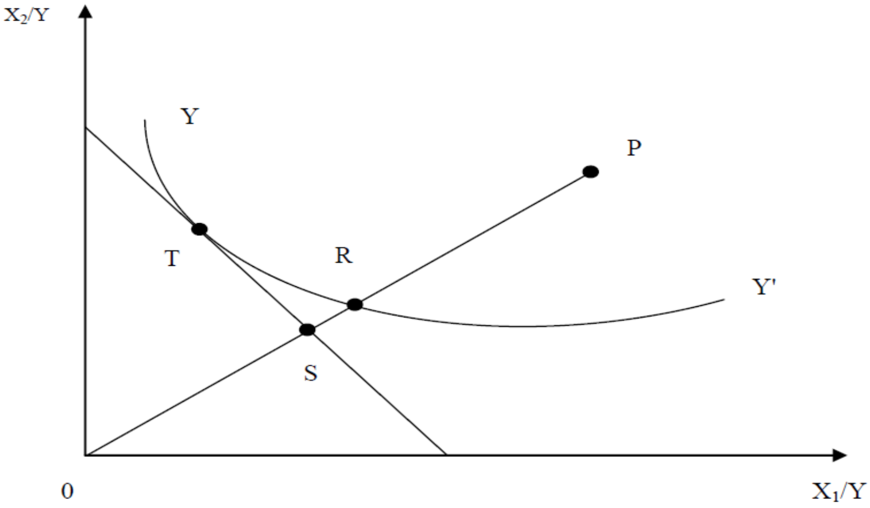
2.1 The Concept of Efficiency

The primary motive of producers is to maximize their output levels subject to available inputs. However, this objective is not always achieved and in most cases, producers operate below their optimal capacity, given the technology at their disposal. In this regard the use of conventional production functions in solving the optimization problem may be less desirable to the frontier approach. While the production function approach seeks to intersect data of decision making units (DMU), the frontier based approach seeks to envelop data of DMUs. The basic idea of the frontier approach is to provide a numerical evaluation of the performance of a certain number of DMUs from the perspective of technical efficiency; which is their ability to operate close to or on the boundary of their production set (Daraio and Simar, 2007).

Farrell (1957) is credited with the earliest attempt to provide a generally acceptable measure of efficiency. Efficiency of any decision making unit, as noted by Farrell (1957) basically means the success of the unit to produce the largest possible output from the inputs available. The overall efficiency of a DMU can be defined as the product of two distinctive measures of efficiency namely; technical and price efficiency. A DMU is considered to be technically efficient when it uses fewer inputs to achieve a given level of output or more outputs with a given amount of inputs. The price efficiency on the other hand measures the extent to which a DMU uses the various factors of production in the best proportions, in view of their prices. The resulting inefficiency arising after controlling for input prices are also known as allocative inefficiency (Herrera and Pang, 2005).

An illustration of the two types of efficiency mentioned above are presented in the figure below, following Farrell (1957) and Herrera and Pang (2005). The starting point is to define an isoquant curve YY' that depicts the set of minimum inputs required for a unit of output. Point P defines an input-output combination which uses input quantities X_1 and X_2 to produce a unit of output. However at point R, it is possible to produce one unit of output using less of both inputs. The level of inefficiency in the use of resources can therefore be described by the segment RP. This type of

Technical efficiency (TE) can be defined as TE=OR/OP. There is also a possibility for the DMU to reduce cost by choosing another input combination. Point P provides such cost reduction option where one unit of output can be produced at the least cost combination of inputs. This is depicted by the equality of the marginal rate of technical substitution and the input price ratio. To achieve this cost level implicit in the optimal combination of inputs, there is the need to contract the input use to point S. The input allocative efficiency (AE) can therefore be defined as AE=OS/OR.



2.2 Empirical Review

Evans et al. (2001) is credited with the first attempt to estimate the efficiency of health systems using a non-parametric approach. Using data from 191 WHO countries (both developed and developing), they estimated the relation between levels of population health and the inputs used to produce health with data from 1993 to 1997. While population health output was measured by healthy life expectancy, health system input was measured by per capita health expenditure. The results showed Oman to be the most efficient with a score of 0.992 and Zimbabwe the least efficient with a score of 0.080. They argued that health system performance was likely to be influenced by civil unrest and the prevalence of HIV and AIDS. In a similar study, Hernandez de Cos and Moral-Benito (2011) used panel data for 29 OECD countries with annual observation from 1997 to 2009. The authors employed a stochastic frontier analysis (SFA) which showed that Japan was the most efficient country in terms of health system performance. The authors also showed that both health system efficiency and health care expenditure positively influence life expectancy with elasticity of 0.71 and 0.06, respectively.

Further, Jayasuriya and Wodon (2003) estimated public sector efficiency in the health and education sectors using SFA. A panel sample of 79 countries over the period 1990-1998 was used in the analysis. While health outcome was proxied by life expectancy, GDP per capita, adult literacy and health expenditure per capita (private and public) were used as input variables. In the second stage analysis the authors conclude that urbanization and bureaucratic quality were significant determinants of efficiency. No conclusive evidence was established for corruption.

In a critique of panel studies that estimated efficiency using the SFA, Greene (2004) provided evidence to show that unobserved heterogeneity can influence efficiency estimates, especially, in cross-country studies and should be accounted for. Failure to treat this may limit the reliability of the efficiency estimates and render them biased. This motivated the objective of the current study to compare estimates from various models including those that treat unobserved cross-country heterogeneity as proposed by Greene (2004).

3. METHODOLOGY

3.1 The Stochastic Frontier Model

The stochastic frontier model is believed to be originally proposed by the works of Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). The model is basically motivated by the idea that deviations from the production 'frontier' might not be entirely under the control of the DMU being studied. For instance, in the case of health systems (which is the focus of the current study) several other factors, such as macroeconomic performance, education etc, may influence efficiency even though they are not under the control of the health system. Also, any error or imperfection in the specification of the model or measurement of its component variables, including the output, could likely translate into increased inefficiency measures. This makes deterministic frontier models unattractive.

While both cross section data and panel data have been used in estimating stochastic frontier models, Belotti et al. (2012) argued that availability of a richer set of information in panel data relaxes some of the assumptions and considers a more realistic characterization of the inefficiencies. The first empirical model using longitudinal data under the SFA is attributed to Pitt and Lee (1981). Their work was based on Maximum Likelihood (ML) estimation of the following Normal-half Normal stochastic frontier model

$$y_{it} = \alpha + x_{it}^{'}\beta + \varepsilon_{it}, \qquad i = 1,....,N \qquad t = 2,...T$$

$$\varepsilon_{it} = v_{it} - u_i$$
$$v_{it} \sim N(0, \delta_v^2)$$
$$u_i \sim N^+(0, \delta_u^2)$$

where y is the output variable and *x* represents a vector of inputs. ε is the error term decomposed into the normal error term (*v*) and inefficiency term (*u*).

In generalizing the above specification, Battese and Coelli (1988) proposed a Normal-Truncated Normal model. In a similar way, Schmidt and Sickles (1984) proposed that fixed effect estimation techniques can be employed to SF models with time invariant inefficiency. This approach enables one to avoid distributional assumptions about u_i. A major limitation of the time invariant models is that the efficiency estimates may be biased in the case of long panel data sets. To resolve this problem, Cornwell et al. (1990) proposed the following SF model with individual-specific slope parameters

$$\begin{aligned} y_{it} &= \alpha + x_{it}^{'}\beta + v_{it} \pm u_{it}, \\ u_{it} &= \omega_i + \omega_{i1}t + \omega_{i2}t^2, \end{aligned} \qquad i = 1,...,N \qquad t = 1,...,T$$

The parameters of this model can be estimated using the conventional fixed and random effects panel data estimators. This specification is limited by its requirement of a large number of parameters. Lee and Schmidt (1993) proposed an alternative specification in which u_{it} is specified as $u_{it} = g(t).u_i$, where g(t) is represented by a set of dummy variables. While this specification is considered to be more parsimonious, it restricts the temporal form of u_{it} to be the same for all DMUs.

Kumbhakar (1990) is considered to be the first to propose the maximum likelihood estimation of a time-varying SF model where $g(t) = [1 + \exp(\gamma t + \delta t^2)]^{-1}$; γ and δ are parameters to be estimated. Battese and Coelli (1992) also proposed a similar model in which $g(t) = \exp[(-\gamma(t - T_i)]$. This model is commonly known as the "time decay" model.

A common feature of the time-varying models is that the intercept (*α*) is the same across DMUs, thus generating a misspecification bias in the case where time-invariant unobservable factors (which may be unrelated with the production process but affecting the output) are available. Such unobservable factors may be captured by the inefficiency term and may lead to biased estimates. Greene (2004) showed that these restrictions can be relaxed by placing country specific constant terms in the stochastic frontier model. This approach is called the 'True' fixed effect model. The specification is given as follows;

$$y_{it} = \alpha_i + x_{it}^{'}\beta + v_{it} - u_{it}$$

The model is estimated using ML and simply involves the inclusion of a full set of country dummy variables in the stochastic frontier model. The model also treats country specific time-invariant fixed effects (α_i) and time varying inefficiency (u_{it}) separately and is therefore able to distinguish between the unobserved heterogeneity and inefficiency (Danqua et al., 2013). The shortcomings of the TFE model include the possibility of incidental parameters problem and over specification of the model with the inclusion of the country specific dummies. An alternative to resolving the unobserved heterogeneity problem is to estimate a time invariant random term meant to capture country specific heterogeneity. This process is termed the 'true' random effect (TRE). The TRE model can be specified as follows;

$$y_{it} = \alpha + \beta^{'} x_{it} + \omega_i + v_{it} - u_{it}$$

Where ω_i is a time-invariant and country specific random term meant to capture unobserved country specific heterogeneity. The model is estimated using the simulated maximum likelihood (SML). As noted by Greene (2004), this form of the model overcomes both of the drawbacks in the TFE specification. The current study estimates an application of the time-varying stochastic frontier models to health system efficiency across SSA countries. The time-varying frontier models employed in the comparative analysis include the Battese and Coelli (BC) model, Kumbhakar (Kumb) model, "true" fixed effect (TFE) and "true" random effect (TRE) models.

An important component of SFA models is the specification of the functional forms of the production function. The Cobb-Douglas specification is the commonly used type in the literature due to its simplicity. However, this specification is restricted in the sense that the return to scale takes the same value across all DMUs in the sample and elasticities of substitution are assumed to be equal to one. In view of this limitation, several alternative specifications of the functional form has been suggested in the literature. The most notable include the translog specification (Greene, 1980b) and the Zellner-Revanker generalised production function (Forsund and Hjalmarsson, 1979, Kumbhakar *et al.*, 1991). While the later specification removes the returns to scale restriction, the former imposes no restrictions on returns to scale or substitution possibilities. The Cobb-Douglas functional type has, however, been confirmed to be a sufficient functional form specification of stochastic frontier production functions.

3.2 Data and Variables

Data for the study was obtained from the World Bank world development indicators. The data covered the period 1995 to 2010 across 45 countries in SSA[1]. This suggests that a total panel sample of 315 was used in the analysis. The use of 45 countries grants added advantage to the study as relatively better efficiency estimates are obtained from larger observations with the frontier methodology. The dependent variable or health system outputs used in the efficiency analysis was infant mortality rate. However, as noted by Afonso and Aubyn (2005), efficiency measurement techniques suggest that outputs are measured in such a way that "more is better". Therefore consistent with practice in the literature, the following transformation was performed on the mortality variable so that it is measured in survival rate. Thus, infant mortality rate (IMR) was measured as

[(number of children who died before 12 months)/(number of children born)] X 1000

This implies that an infant survival rate (ISR) can be computed as follows;

$$ISR = \frac{1000 - IMR}{IMR}$$

This shows the ratio of children that survived the first year to the number of children that died and this increases with better health status. In the case of the independent variables (health system inputs), monetary input was used instead of physical input as this is considered as a broader measure of health system input. Per capita health expenditure was used as the main input variable that directly influences the health system. Other indirect inputs used in the study that influence the performance of the health system but lies beyond its control include HIV/AIDS, education, per capita gross domestic product (GDP).

Table 1: Summary of variable description and data source

Variables	Description	Data source
Infant mortality rate (IMR)	The probability of a child born in a specific year or period dying before reaching the age of one	World Development Indicators (WDI)
Per capita health care expenditure (HCEpc)	Per capita total expenditure on health expressed in purchasing power parity (ppp) international dollar	WDI
Public health care expenditure (PuHE)	Level of public spending on health as percent of total government spending. Includes spending from government budgets, external borrowing, grants and social health insurance funds	WDI
Real GDP per capita (GDPpc)	Real GDP per capita measured in constant 2005 international dollars	WDI

[1] The following countries were included in the study: Angola, Benin, Burkina Faso, Botswana, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo Demographic Republic, Congo, Cote d'Ivoire, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Ghana, Guinea, Guinea Bissau, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Mozambique, Namibia, Niger, Nigeria, Rwanda, South Africa, Sao Tome, Senegal, Seychelles, Sierra Leone, Sudan, Swaziland, Tanzania, The Gambia, Togo, Uganda and Zambia.

Education (Educ.)	Secondary school enrolment as percentage of gross school enrolment	WDI
HIV prevalence rate (HIV)	Estimated number of adults aged 15-49 years with HIV infection expressed as percent of total population in that age group	WDI

Source: Authors' compilation

4. RESULTS AND DISCUSSION

4.1 Descriptive Statistics

Table 2 shows summary statistics for the variables included in the analysis. As mentioned earlier, the rate of under five survival was used as the outcome variable instead of the usual under five mortality rate. The summary statistics show that on average, the ratio of children that survived the first year to the number of children that died was about 12.40 with minimum and maximum values of about 3.66 and 71.46, respectively. Average health care expenditure per capita (HCEpc) was about US$186.38. On average, secondary school enrolment as percentage of gross school enrolment was about 32.38% with a minimum of 1% and a maximum of 72%. Average HIV prevalence rate among adults was 5.26% with a minimum of 0.1% and maximum of 26%.

Average gross domestic product per capita (GDPpc) over the sample period was US$3387.56. The minimum GDPpc over the sample period was US$294.39 while a maximum of about US$27346.4 was recorded. In terms of government commitment to the health sector, the statistics show that public health care expenditure as percentage of total government spending recorded an average of about 3.80% over the sample period, with a minimum of about 2.45% and a maximum of about 4.54%.

Table 2: Descriptive Statistics

Variable	Obs.	Mean	Std. Dev.	Minimum	Maximum
USR	315	12.3954	13.5249	3.6642	71.4638
HCEpc	315	186.3835	257.8733	14.5853	1806.481
Educ	315	32.3835	19.8794	1	72
HCEpub	315	3.7995	0.4158	2.4505	4.5376
GDPpc	315	3387.555	5248.812	294.3864	27346.4
HIV	301	5.2558	6.5791	0.1	26

Source: Authors' compilation

4.2 Estimated Stochastic Frontier

The estimated production frontier functions for the SFA efficiency and inefficiency estimates are presented in Table 3 below. The table shows estimates for all the time varying specifications discussed earlier. The results show that the necessary estimated parameters that give indications about the reliability of the efficiency estimates are acceptable for all the specifications. For instance, the estimate of λ was statistically significant for all the specifications and this confirms that there is the existence of technical inefficiency in the dataset. The value of λ is also smallest for the TRE model specification relative to all the other model specifications. On the other hand, the TFE model specification recorded the highest value of λ. The important indication, however, lies in the statistical significance of the parameter.

Table 3: Estimated stochastic frontier models

Models	Battese and Coelli	Kumbhakar	'True' Fixed Effect	'True' Random Effect
lnHCEpc	-0.81842 (0.83520)	0.33078*(0.17744)	0.15073***(0.00008)	0.16522***(0.06194)
lnHCEpc2	0.11058 (0.09272)	-0.03208**(0.01475)		-0.00275(0.00541)
lnEduc	-0.04459 (0.10318)	-0.01716***(0.00656)	-0.02889***(0.00002)	-0.02672***(0.00367)
lnHCEpub	0.42274***(0.16069)	0.01796 (003931)		0.03718 (0.02598)
lnGDPpc	0.12299 (0.16940)	0.48267*** (0.15423)	0.33889***(0.00005)	0.25879*** (0.01784)
lnHIV			-0.12909***(0.00003)	-0.23468***(0.00713)
Constant	1.48291(2.17289)	-1.08345 (1.45488)	Na	-0.13801(0.14071)
Λ	9.81331***	37.24355***	20.905***	3.76068***
Δu	3.70134	2.59807***	0.08907***	0.09024***
Δv	0.37718***	0.06976***	4.26E+08	0.0240***
Δw	Na	Na	Na	0.58418***

Source: Authors' computation Note: 1. Robust standard errors are reported in parenthesis.
2. ***significant at 1%; **significant at 5%; *significant at 10%.

As expected, the variance decomposition was dominated by u (δ_u). The TFE and the TRE had the lowest values of the δ_u relative to the Battese and Coelli as well as Kumbhakar specifications while δ_v is almost similar for all the specification except for the TFE models which is relatively smaller. The component of the variance ($\delta_{w)}$) introduced in the TRE model to control for unobserved heterogeneity among cross section units was statistically significant. This suggests that the TRE model specification actually purges u_i of time invariant heterogeneity.

4.3 Estimated Mean Efficiency Scores in SSA

Table 4 below provides summary statistics of the estimated efficiency scores from the various model specifications employed in the study. The table shows summary statistics on the mean, standard deviation as well as minimum and maximum values over the period 2005-2011. The lowest average health system efficiency score was recorded for the Kumbhakar SFA model specification. While the other three model specification showed similar mean estimates of health system efficiency, the highest estimate was recorded for the TFE specification. In sum, the Battese and Coelli specification recorded an average efficiency score of about 0.76 for health systems in SSA over the period 2005-2011, with minimum and maximum values of about 0.15 and 0.91, respectively. In a significant deviation from the BC model, the Kumbhakar model recorded average efficiency score of about 0.38 with minimum and maximum values of 0.06 and 0.94, respectively. The TFE model recorded the highest mean efficiency score of about 0.92 while the TRE model recorded an average efficiency score of about 0.80.

Table 4: Summary of mean efficiency scores (2005-2011)

Models	Mean	Standard Deviation	Minimum	Maximum
Battese and Coelli	0.76304	0.12192	0.14608	0.91489
Kumbhakar	0.38315	0.20522	0.06203	0.94470
'True' Fixed Effect (TFE)	0.91966	0.08928	0.58254	0.99999
'True' Random Effect	080255	0.21443	0.26464	0.99936

Source: Author's computation

In Table 5, the average efficiency scores for the individual countries included in the analysis are presented. The results show that the rankings of countries and mean health system efficiency score varied significantly across countries. There was however significantly high variation between the ranks produced by the TFE model specifications and the other three specifications. For instance Mauritius was estimated to be, on average, the most efficient health system in the regions from the Battese and Coelli as well as the TRE models. However, in the TFE model, Mauritius was ranked 21. Similarly, health system efficiency in Cape Verde was estimated and ranked to be 1 in the Kumbhakar model and 2 in the BC and TRE models. Cape Verde was however ranked 22 in the TFE model.

A sharp contrast is also observed between the models in terms of the worst performing countries' health systems. While such countries as Angola was unanimously estimated to be relatively less efficient in the Battese and Coelli, Kumbhakar and TRE models, the rank of South Africa was different between the models. While South Africa was ranked to be 41, 33 and 42 in the Battese and Coelli, Kumbhakar and TFE models, respectively, the country's health system was ranked 17 based on the TRE model. Also, a country like Malawi was ranked 21 and 36 based on the Battese and Coelli and TFE models, respectively, while it ranked 4 based on the TRE model.

The distinction between the different model specifications may be attributed to the time invariant unobserved heterogeneity present in the panel data. This justification supports the TRE as the preferred model due to its ability to accommodate this limitation and produce efficiency estimates based on pure technical efficiency. In this regard the empirical evidence based on the TRE model specifications suggest that Mauritius, Cape Verde, Botswana, Malawi and Tanzania have the best five performing health systems in SSA while countries like Angola, Sierra Leone, Equatorial Guinea and Mali have the relatively worst performing health systems.

4.4 Correlation between Time Varying Frontier Models

In Table 6 below a further comparison of the various model specifications was performed to ascertain the correlation between the models. The simple correlation coefficients reported in the table suggest that there is some similarity between the Battese and Coelli and the Kumbhakar model specifications while a weak relationship was established between these models and 'True' fixed and random effect models. For instance, while the correlation coefficient between the Battese and Coelli model and the Kumbhakar model was about 76%, the coefficients between these two and the TRE model was about -4% and -2%, respectively. The correlation coefficient also showed above average (about 68%) relationship between the TFE and TRE models. In sum the simple correlation matrix suggest stronger similarity between the models[2] that do not accommodate any time invariant unobserved heterogeneity and the models that accommodate this limitation of panel data analysis, as in the present study[3].

[2] In this case the models are the Battese and Coelli and the Kumbhakar models
[3] Such models in the case of the current study are the 'True' fixed and random effect models

Table 5: Ranks and Estimated Mean Efficiency Scores (2005-2011)

DMU	Rank	BC	Rank	Kumb	Rank	TFE	Rank	TRE
Angola	44	0.457033	44	0.106905	29	0.916133	42	0.3128233
Benin	26	0.778363	27	0.30857	18	0.940387	31	0.6860212
Botswana	25	0.779231	15	0.411364	40	0.824088	3	0.999087
Burkina Faso	40	0.642914	35	0.223825	6	0.981737	39	0.5285583
Burundi	24	0.785182	18	0.389762	4	0.986021	24	0.8232362
Cameroon	23	0.788912	37	0.21889	1	0.995963	26	0.7737165
Cape Verde	2	0.904113	1	0.941164	22	0.929331	2	0.9992674
Central African Rep.	39	0.691309	30	0.289741	5	0.982737	20	0.906207
Chad	37	0.712582	40	0.192761	15	0.963657	38	0.5715332
Comoros	11	0.840136	9	0.477892	19	0.938188	35	0.6129595
Congo, Dem. Rep.	32	0.74398	14	0.42144	44		44	
Congo, Rep.	28	0.76612	36	0.219852	13	0.967862	28	0.7359002
Cote d'Ivoire	17	0.816553	31	0.254572	7	0.976998	25	0.7851072
Equatorial Guinea	45	0.255202	45	0.071679	32	0.901384	43	0.2828663
Eritrea	6	0.857402	3	0.876949	23	0.925097	7	0.9986047
Ethiopia	16	0.828099	8	0.532453	12	0.969542	15	0.9972692
Gabon	38	0.710676	42	0.176608	17	0.959882	27	0.7547606
Gambia, The	27	0.773385	29	0.289779	20	0.937681	30	0.7069977
Ghana	13	0.833844	13	0.43151	8	0.976177	16	0.9952689
Guinea	19	0.806623	28	0.298199	28	0.917325	29	0.7202048
Guinea-Bissau	31	0.753172	34	0.227852	9	0.971514	36	0.6107064
Kenya	7	0.85605	12	0.431746	31	0.905901	6	0.9986509
Lesotho	29	0.760638	26	0.327941	24	0.923763	9	0.9983131
Liberia	4	0.873906	6	0.670431	3	0.986374	12	0.9978673
Madagascar	5	0.862967	5	0.672711	35	0.866919	19	0.9835736
Malawi	21	0.789592	11	0.465731	36	0.849654	4	0.998837
Mali	42	0.617029	39	0.19332	11	0.970026	40	0.4312364
Mauritania	33	0.73448	32	0.245103	10	0.970882	37	0.5776037
Mauritius	1	0.908957	2	0.933838	21	0.932324	1	0.9993186
Mozambique	30	0.759222	21	0.377901	25	0.920953	13	0.9978106
Namibia	20	0.799704	25	0.341099	41	0.782302	8	0.9985626
Niger	35	0.722628	22	0.351006	38	0.8273	34	0.6246859
Nigeria	34	0.732946	38	0.202217	26	0.920564	32	0.6446696
Rwanda	9	0.844799	7	0.546375	42	0.780251	10	0.9982338
Sao Tome	14	0.832635	20	0.378134	16	0.960458	23	0.8370534
Senegal	15	0.829727	16	0.408174	39	0.824521	21	0.8707598
Seychelles	3	0.882656	4	0.799165	45		45	
Sierra Leone	36	0.71439	41	0.187952	14	0.967419	41	0.3776862
South Africa	41	0.639944	33	0.243969	43	0.755632	17	0.9894383
Sudan	12	0.834039	23	0.34665	30	0.915588	33	0.6318157
Swaziland	43	0.514951	43	0.164005	34	0.889071	22	0.854706
Tanzania	10	0.841992	10	0.469514	33	0.895339	5	0.9988337
Togo	18	0.816171	19	0.385826	2	0.987012	18	0.9877306
Uganda	8	0.853092	17	0.392145	27	0.92022	14	0.9973324
Zambia	22	0.789507	24	0.34503	37	0.831325	11	0.9978735
Mean		**0.76304**		**0.38315**		**0.91966**		**0.8045**

Source: Author's computation

The scatter plots in Figure 1 provide further evidence on the correlation between the various model specifications. From the first plot, the figure shows correlation between the time varying Kumbhakar model and the time invariant Battese and Coelli model. There was evidence of strong correlation between the two models and this implies some similarity between the Kumbhakar model[4] and the time invariant model.

Table 6: Correlation between inefficiency estimates

Models	Battese and Coelli	Kumbhakar	'True' Fixed Effect	'True' Random Effect
Battese and Coelli	1.0000			
Kumbhakar	0.7645	1.0000		
'True' Fixed Effect (TFE)	0.1061	-0.0649	1.0000	
'True' Random Effect	-0.0437	-0.1861	0.6785	1.0000

Source: Author's computation

The next plot on the right shows some correlation between the two time varying models that accommodate unobserved heterogeneity, in this case, the 'true' fixed and random effect models. Evidence from the two plots at the bottom of Figure 1 also suggests that there is very weak correlation between the time varying Battese and Coelli, Kumbhakar models and the 'true' random effects model.

Figure 1: Scatter plot of inefficiency scores form time varying frontier models

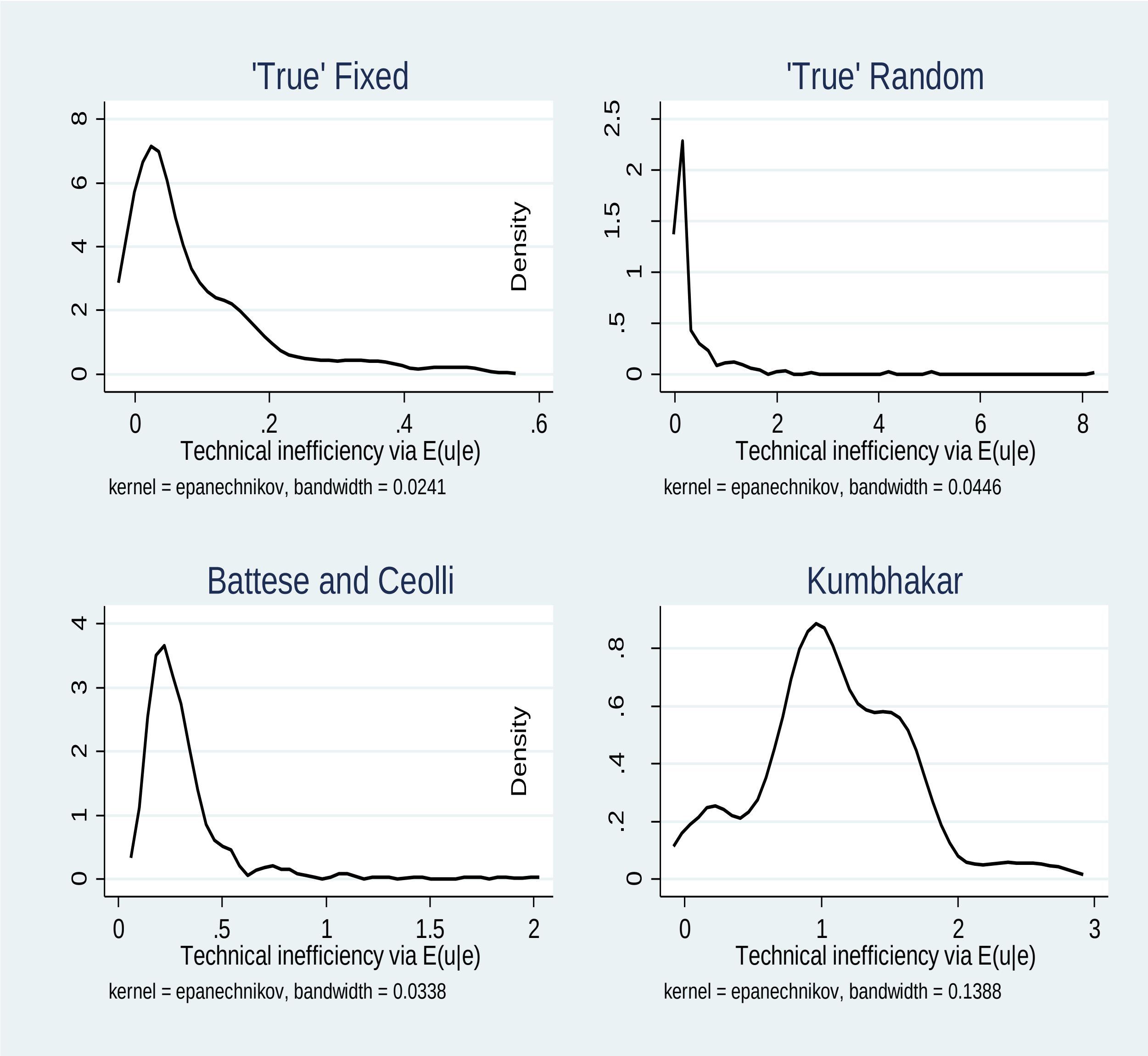
Source: Authors' computation

This implies that the TRE model specification which attempts to sluice unobserved heterogeneity from the inefficiency term (u_{it}) produces unique estimates of inefficiency and is unrelated to other time varying models that do not

[4] It must be recalled that the Kumbhakar model is a time varying model as discussed earlier in the study.

account for unobserved heterogeneity. This makes the TRE a more preferred model specification in stochastic frontier analysis using panel data.

Figure 2: Kernel Density Estimates of inefficiency scores from time varying frontier models



Source: Authors' computation

In Figure 2 below the mean and variation of the distributions of the estimated inefficiencies from the various model specifications are analysed using the kernel density estimates. The results from the kernel density estimates show that the 'true' random effects model has the lowest mean relative to all the other time varying model specifications. Also the variance of the 'true random effect model is significantly lower compared to the other models.

Similarly, the 'true' fixed effect model has lower mean compared to the Battese and Coelli and the Kumbhakar models, even though the variance of the Battese and Coelli seems to be marginally lower than the 'true' fixed effect model. The Kumbhakar model specification had the largest mean and variance from the kernel density estimators. This implies that the Kumbhakar model was the most dispersed in terms of the distribution of the means and variance. Again this confirms the suitability of the 'true' random effect model specification, compared to the other time varying models. The distribution of the mean and variance of the inefficiency term (u_{it}) of the 'true' random effect model was much lower and tighter than the other models. This is an addendum to the earlier evidence that the 'true' random effects model effectively deals with the time invariant heterogeneity present in panel data modelling of SFA.

5. CONCLUSION

The study set out to estimate health system efficiency across countries in SSA using various time varying stochastic frontier models. The study employed panel data between the period from 1995 to 2011 over 45 countries. Four main time varying stochastic frontier models were analysed in the study. These models include the Battese and Coelli model, the Kumbhakar model, the 'True' fixed effect model and the 'True' random effect model. The 'true' random and fixed effects models were estimated using the Simulated Maximum Likelihood (SML) while the Battese and Coelli and the Kumbhakar models were estimated using the Maximum Likelihood (ML) technique. The 'true' random and fixed effects models were also based on the 'Exponential' distributional assumption while the Battese and Coelli model was based on the 'Truncated Normal' and the Kumbhakar model based on the 'Half Normal' distributional assumption.

Based on the various empirical analysis in the study, the 'true' random effect model appeared to be the most preferred considering its ability to control for time invariant unobserved heterogeneity across decision making units. The inability of the Battese and Coelli model and the Kumbhakar models to treat unobserved heterogeneity translated into the estimated inefficiency scores from these models. Strong correlation was established between these two models and also with some time invariant stochastic frontier models.

Based on the 'true' random effect model, the study showed that average health system efficiency over the period 2005-2011 was about 80%. This implies that health systems in the SSA region were less efficient in the use of health system resources. The statistics implies that on average, SSA countries have the potential to improve health system efficiency by about 20%. The findings also showed that on average, given the level of health system resources in SSA, health outcomes[5] can be improved by about 20% if these resources are used efficiently. With regards to the individual country efficiency analysis, the results showed that countries like Mauritius, Cape Verde, Malawi, Botswana and Tanzania were estimated to be, relatively, the most efficient countries. On the other hand, Equatorial Guinea, Sierra Leone, Mali and Angola were among the poor performers in terms of health system efficiency.

The findings of the study call for critical policy efforts to improve the performance of health systems in developing countries, especially SSA countries. This implies that while higher levels of health care spending is important for mitigating the huge burden of health care in SSA, the efficiency in the use of these resources is equally important. That is to say that, increasing health system spending is a necessary condition but a sufficient condition will be to spend these resources in an efficient way. In shifting policy focus to the post 2015 agenda, improving health system efficiency should be an important component.

6. REFERENCES

Afonso, A. and M. S. Aubyn (2005) 'Non-parametric approaches to education and health efficiency in OECD countries', *Journal of Applied Economics*, 8(2), 227-246.

Aigner, D., C. A. K. Lovell and P. Schmidt (1977) 'Formulation and estimation of stochastic frontier production function models', *Journal of Econometrics*, 6(1), 21-37.

Battese, G. and T. Coelli (1992) 'Frontier production functions, technical efficiency and panel data: with application to paddy farmers in India', *Journal of Productivity Analysis*, 3, 153-169.

Battese, G. E. and T. J. Coelli (1988) 'Prediction of firm-level technical efficiencies with a generalised frontier production function and panel data ', *Journal of Econometrics*, 38, 387-399.

Belotti, F., S. Daidone, G. Ilardi and V. Atella (2012) 'Stochastic frontier analysis using Stata', *Center for Economic and International Studies Tor Vergata Research Paper Series*, 10(12), 1-48.

Cornwell, C., P. Schmidt and R. C. Sickles (1990) 'Production frontiers with cross-sectional and time series variation in efficiency levels', *Journal of Econometrics*, 46, 185-200.

Danqua, M., A. Barimah and O. Williams. (2013) 'Efficiency measurment using a "true" random effects and random parameter stochastic frontier models: an application to rural and community banks in Ghana', *Modern Economy*, 4, 864-870.

Evans, B. D., A. Tandon, L. J. C. Murray and A. J. Lauer (2001) 'Comparative efficiency of national health systems: cross national econometric analysis', *British Medical Journal*, 323, 307-310.

Farrell, M. (1957) 'The measurement of productive efficiency', *Journal of the Royal Statistical Society. Series A (Statistics in Society)*, 120(3), 253-290.

[5] It should be recalled that infant mortality rate was transformed into infant survival rate for the current study

Forsund, F. R. and L. Hjalmarsson (1979) 'Generalized Farell measures of efficiency: an application to milk processing in Swedish diary plants', *Economic Journal,* 89, 294-315.

Greene, W. (1980b) 'On the estimation of a flexible frontier production model', *Journal of Econometrics,* 13, 101-116.

Greene, W. (2004) 'Distinguishing between heterogeneity and inefficiency: stochastic frontier analysis of World Health Organization's panel data on national health care systems ', *Health Economics,* 13, 959-980.

Hakkinen, U. and I. Joumard (2007) 'Cross-country analysis of efficiency in OECD health care sectors: options for research ', *OECD Economics Department Working Papers No. 554*.

Hernandez de Cos, P. and E. Moral-Benito (2011) 'Health ecpenditure in the OECD countries: efficiency and regulation', *Bank of Spain Occasional Documents No. 1107*.

Herrera, S. and G. Pang (2005) 'Efficiency of public spending in developing countries: an efficiency frontier approach', *World Bank Policy Research Working Paper 3645*.

Jayasuriya, R. and Q. Wodon (2003) 'Efficiency in reaching the millennium development goals', *World Bank Working Paper No.9*.

Kumbhakar, S. C. (1990) 'Production frontiers, panel data and time varying technical inefficiency', *Journal of Econometrics,* 46, 201-211.

Kumbhakar, S. C., S. C. Ghosh and J. T. McGuckin (1991) 'A generalised production frontier approach for estimating determinants of inefficiency in U.S. diary farms', *Journal of Business & Economic Statistics,* 9, 279-286.

Lee, L. F. and P. Schmidt (1993) *The measurement of productivity efficiency: techniques and applications, chap., A production frontier model with flexible temporal variation in technical inefficiency,* Oxford: Oxford University Press.

Meeusen, W. and J. van den Broeck (1977) 'Efficiency estimation from Cobb-Douglas production function with composed errors', *International Economic Review,* 18(2), 435-444.

Pitt, M. M. and L. F. Lee (1981) 'The measurement and sources of technical inefficiency in the Indonesian weaving industry', *Journal of Development Economics,* 9, 43-64.

Powell-Jackson, T., K. Hanson and D. Mclntyre (2012) 'Fiscal space fro health: a review of the literature', *Resilient and Responsive Health Systems Working Paper No.1*.

Schmidt, P. and R. C. Sickles (1984) 'Productivity frontier and panel data ', *Journal of Business & Economic Statistics,* 2, 367-374.

WHO (2000) *The world health report 2000. Health systems: improving performance,* Geneva: World Health Organization.

WHO (2012) *World health statistics 2012*, Geneva: World Health Organization.

World Bank (2010) *World development indicators*, Washington DC: The World Bank.

World Bank (2012) *World development indicators dataset*, Washington D.C: The World Bank.