Comparative Study of Ultimate Moment and Cost by using ACI 318-08, BS 8110-1:1997 and IS 456:2000

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ABSTRACT— ***When designing a safe and economical structure, the ultimate moment a rectangular beam defines the strength of the structure. By considering the above needs, this study gives the comparative design of ultimate moment of a rectangular beam by using three different international design codes. The reduction factor used differs for different codes which affects the strength of the flexural member. The design data obtained from a particular design is used in other standards to bring out similar results. The design methodology is quite similar, but there are some differences in the parameters and constants mentioned in the three code provisions. The cost is estimated to ensure an economical design.***

Keywords— Ultimate moment, ACI 318-08, BS 8110-1:1997 and IS 456:2000

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1. INTRODUCTION

The study of ultimate moment in a rectangular section by ACI 318-08 and BS 8110-1:1997 are comparatively studied with IS 456:2000. This study is about the economical design and cost of the flexural element. The objective of the study, the constants and parameters used in this paper are discussed in the best way as possible. The design procedure includes brief explanation for the procedure used, parameters and constants that vary according to different standards and elimination of few calculations in various codes of practice. Tables are included briefly to discuss the cost of the element and ultimate moment separately. The cost includes labor cost and material cost which defines the cost efficiency of a section by using the three codes of practice. Eventually the results are evaluated and compared in tabular format.

The paper can be of great value to budding engineers to comparatively learn the design aspects of the international codes - ACI 318-08, BS 8110-1:1997 and IS 456:2000 as it carries an example for each code of practice to explain the research in detail.

2. OBJECTIVE OF STUDY

i. To compare the design strength of the beam for the following three code provisions: ACI 318-08, BS 8110-1:1997 and IS 456:2000.

ii. To comparatively study the design procedure for beams of the following codes of practice— ACI 318-08, BS 8110-1:1997 and IS 456:2000.

3. DESIGN PROCEDURE FOR ACI 318-08, BS 8110-1:1997 AND IS 456:2000

Table 1: Comparison of design procedures

S.NO	DESCRIPTION	ACI 318-08	BS 8110-1:1997	IS 456:2000
1.	Factored load: Bars in compression are near the neutral axis, hence a factor is assumed to be greater than 3%.	Find the factored loads, $W_u = 1.2W_D + 1.6W_L$	W= 1.4 g_k + 1.6 q_k	Area of reinforcement is taken from previous method; hence load calculation is not needed.
2.	Compression and tension forces: Axial tension and flexure is taken as 0.95. In IS 456 steel yield is assumed to be 80%.	Check for minimum steel compromises with this calculation.	Equilibrium of the compressive and tensile forces. $0.95f_yA_s = 0.45f_{cu}b(0.9x)$ Where, $x = 2.346 A_sf_y/f_{cu}b$	Forces of compression = Forces of tension C = T $0.36f_{ck}bd = 0.87f_yA_{st}$
3.	Lever arm: Lever arm is the distance between force of compression and tension in a beam.	The lever arm between tensile and compressive resultants in a concrete beam is equal to 90% of the effective depth (*d*), then the (*d-a/*2) component of the "exact" equation above simply becomes 0.9d.	Lever arm, z = d-0.45x $z = d(1- 1.056 A_sf_y/f_{cu}b)$	Lever arm, z = d-0.42x $z = d- (f_yA_{st}/f_{ck}b)$
4.	Percentage steel	$M_{u\ dist} = W_uL^2/8$ $P_u = 1.6P$ $M_{u\ point} = P_uL/4$ $M_u = M_{u\ dist} + M_{u\ point}$ Check for minimum steel- $\rho = A_s/bd$ $A_{s\ min}= 3\sqrt{f_c'}b_wd/f_y < A_s$ Assume steel has yielded. C = T $T = f_yA_s$; $C = 0.85f_c'ab$ Check for minimum steel strain- $M_n = T(d- a/2)$	$M_u = 0.95f_yA_sz$ $M_u = 0.95f_yA_s d(1- 1.056 A_sf_y/f_{cu}b)$	M_u (concrete) = $0.36f_{ck}bz$ M_u (steel) = $0.87f_yA_{st}z$
5.	Factor of depth		Factor of depth, k $k = M_u/f_ubd^2$;$\rho = 100A_s/bd$; $m = f_y/f_{cu}$ $k= 0.0095m\rho- 0.0001(m\rho)^2$	Maximum depth of neutral axis $X_m/d=0.0035/(0.87f_y/E_s)+0.0055$
6.	Ultimate moment	Check: $\Phi M_n \geq M_u$	$M = 100A_sf_y/bdf_{cu}$	$M_u=0.87f_yA_{st}d (1-A_{st}f_y / bd^2f_{cu})$
7.	Design stage	Designed at the stage of collapse.	Analysis and fatigue load calculation.	Designed at the point of ultimate strength.

4. METHODOLOGY

4.1 American Code of Practice— ACI 318-08

In this paper we find the ultimate load by determining the self-weight and live load of the beam. Maximum moment is calculated at a distance from the midspan where it occurs. Maximum factored load is assumed as per ACI 318-08 and maximum factored moment is obtained. A method called weighted average is used to determine the depth of the section, d. Area of reinforcement, A_S and minimum area of reinforcement, $A_{S\ min}$ are determined to calculate the tension and compression by assuming that the steel as yielded, in order to check the section for minimum steel and tension control. The beam is checked for adequacy and strength.

4.2 British Code of Practice— BS 8110-1:1997

The ultimate load of the simply supported rectangular beam is calculated by adding the characteristic dead loads and live loads to the constants. The beam is checked for the condition of singly reinforced design by determining the design moment and ultimate moment of resistance. The factor of depth and lever arm is calculated to check the beam for safety. The area of reinforcement is also obtained.

4.3 Indian Code of Practice— IS 456:2000

The rectangular beam section is assumed to be a balanced section to initiate the design; the depth of neutral axis for balanced failure is calculated. The balanced percentage of steel is obtained by equating the compression and tension. It is clear that steel failure controls the design and hence the beam is an under-reinforced section. According to IS 456:2000, Annex G the ultimate moment of resistance for steel is evaluated.

5. DESIGN EXAMPLE

5.1 American Method (ACI 318-08)

In an office building, the point load is a live load with P = 15 kip. The distributed loads are W_d = 0.6638 kip/ft and W_l = 1.45 kip/ft. The span length is L = 26.246 ft. Material strength are f_c' = 4 kip/in^2, f_y = 60 kip/in^2. Is the beam adequate?

Solution:

By equation 9.2. in clause 9.2.1. of ACI 318-08
$W_u = 1.2W_D + 1.6W_L$
$= 1.2(0.6638) + 1.6(1.45)$
$W_u = 3.116$ kip/ft

For a constantly distributed load, the maximum moment occurs at midspan, $M_{u\ dist}$

$M_{u\ dist} = W_uL^2/8$

$= 3.116\ (26.246)^2 = 268.3$ kip. ft

To get the maximum factored load, use $P_u = 1.6P$ in place of P_u.

$M_{u\ point} = P_uL/4$

$= (1.6P)L/4 = ((1.6)(15)(26.246)) / 4$
$= 157.476$ kip. ft
The total factored moment is $M_u = M_{u\ dist} + M_{u\ point}$
$= 268.3 + 157.47$
$= 425$ kip. ft

Weighted average is used to determine d.
d = (((4)(27”)) + ((2)(25”))) / 6 = 26.3”

Now A_s is calculated, $\rho = A_s/bd$

Where $\rho = 0.0239$ (Table from ACI 318)
$0.0239 = A_s$ / ((26.3”)(20”))

$A_s = 1.25$ in^2
$A_s = 6(1.25) = 7.54$ in^2
$A_{s\ min} = 3\sqrt{f_c'}b_wd/f_y$
$= 3\sqrt{4000}$ (20”)(26.3”) / 60000
$= 1.66$ in$^2 < A_s$
$200b_wd / f_y$ = ((200)(20”)(26.3”)) / 60000 = 1.75 in$^2 < A_s$

The steel area A_s which is provided is greater than the above required minimum areas of reinforcement from ACI 318-08.
Assume the steel as yielded.

$T = f_y A_s = (60)(7.54) = 452.4$ kip
By equilibrium, $C = T$
$C = 0.85 f_c' ab$
$a = C / 0.85 f_c' b = T / 0.85 f_c' b$
$= 452.4 / (0.85)(4)(20")$
$a = 6.65"$
For 4000psi $= f_c'$, $\beta_1 = 0.85$
$c = a / \beta_1 = 6.65" / 0.85 = 7.82"$
Check for minimum steel, $c / d_t = 7.82" / 27" = 0.289" \leq 0.375"$

The section is tension controlled. Thus the steel has yielded as assumed and the section meets minimum steel strain requirements of the code.
$\Phi = 0.9$ for a tension controlled section.
$M_n = T (d - a/2)$
$= 452.4 (26.3" - 6.65"/2)(1 \text{ ft}/12 \text{ in})$
$= 866.15$kip.ft
$\Phi M_n = 0.9(866.15") = 780 \text{ kip.ft} \geq M_u$

The beam is adequate in strength and okay.

5.2 British Method (BS 8110-1:1997)

A simply supported rectangular beam 8m span carries a characteristic dead load g_k and imposed loads q_k of 10kN/m and 20kN/m respectively. The beam dimensions are breadth, b = 500mm and effective depth, d = 780mm. Assuming the following material strength: $f_{cu} = 25$ N/mm^2; $f_y = 415$ N/mm^2.

Solution:

Ultimate Load (W) $= 1.4 g_k + 1.6 q_k$
$= (1.4*10) + (1.6*20)$
$= 46$ kN/m

Design method, M $= Wl^2/8$
$= ((46)(8^2)) /8$
$= 368$ kNm

Ultimate moment of resistance, $M_u = 0.156 f_{cu} bd^2$
$= 0.156*25*500*780^2*10^{-6} = 1186.38$ kNm
Hence $M_u > M$, design as singly-reinforced beam.

$k = M / f_u bd^2$

$= 368*10^6 / 25*500*780^2$
$= 0.0483$

Lever arm, $z = d (0.5 + \sqrt{(0.25 - k/0.9)})$
$= 780 (0.5 + \sqrt{(0.25 - 0.0483/0.9)})$
$= 780 (0.942) = 735.52$mm
735.52mm $\leq 0.95d = 741$mm

Hence okay.

Area of steel reinforcement, $A_s = M/0.87 f_y z$
$= 368*10^6 / 0.87*415*735.52 = 1385.75$mm^2

Provide 5 numbers of 20mm diameter bars as longitudinal reinforcement.

5.3 Indian Method (IS 456:2000)

Calculate the Ultimate Moment, in a office building. A rectangular beam is designed for b = 500mm and d = 780mm. The area of steel reinforcement is $A_{st} = 4865$mm^2. Assume grade 25 concrete and Fe 415 steel.

Solution:
Depth of neutral axis for balanced failure $x/d = 0.0035/0.0035 + (0.87(f_y / E_s) + 0.002)$
$E_s = 2*10^5$ N/mm^2
$x / d = 0.0035/ 0.0035+0.00038$
$x / d = 0.5$

Balanced percentage of steel,

$0.87\ f_y\ A_s = 0.36\ f_{ck}\ b_x$

$(P/100\ bd)\ (0.87\ f_y) = 0.36\ f_{ck}\ b_x$

Substituting for x/d, we get

$Pf_y / f_{ck} = 0.36*0.50*100/0.87 = 20.68$

For $f_y = 415 N/mm^2$; $f_{ck} = 25\ N/mm^2$

P = 20.68*25/415 = 1.25%

Actual percentage of steel in beam = 4865*100 /500*780 = 1.2%
Steel failure controls.
Hence, beam is under-reinforced.

M_u steel failure,

$M_u = 0.87 f_y\ (p/100)\ (1-(p/100)\ (f_y / f_{ck}))\ bd^2$

$= 0.87*415*1.2 /100\ (1- 1.2*415 / 100*25)\ *500*780^2$

M_{us} = 1055 kNm

M_u concrete (for conformation only)

$M_u = 0.138\ f_{ck}\ bd^2$

$= 0.138*25*500*780^2$

M_{uc} = 17.42 MNm

Steel failure controls strength, therefore ultimate moment capacity is M_{us} = 1055 kNm

6. RESULT AND DISCUSSION

The Ultimate Moment of a singly reinforced beam of dimension 500mm * 780 mm is calculated by ACI 318-08, BS 8110-1:1997 and IS 456:2000. The cost for the section 500mm * 780mm is also determined in the table 2 shown below.

Table 2: Results by ACI 318-08, BS 8110-1:1997 and IS 456:2000

Code of Practice	Ultimate Moment	Cost of a section
ACI 318-08	1057.5 kNm	174550.73 INR
BS 8110-1:1997	1187 kNm	3838.27 INR
IS 456:2000	1055 kNm	717.6 INR

Chart 1: Cost Analysis of a section 500mm*780mm by ACI 318-08, BS 8110-1:1997 and IS 456:2000.

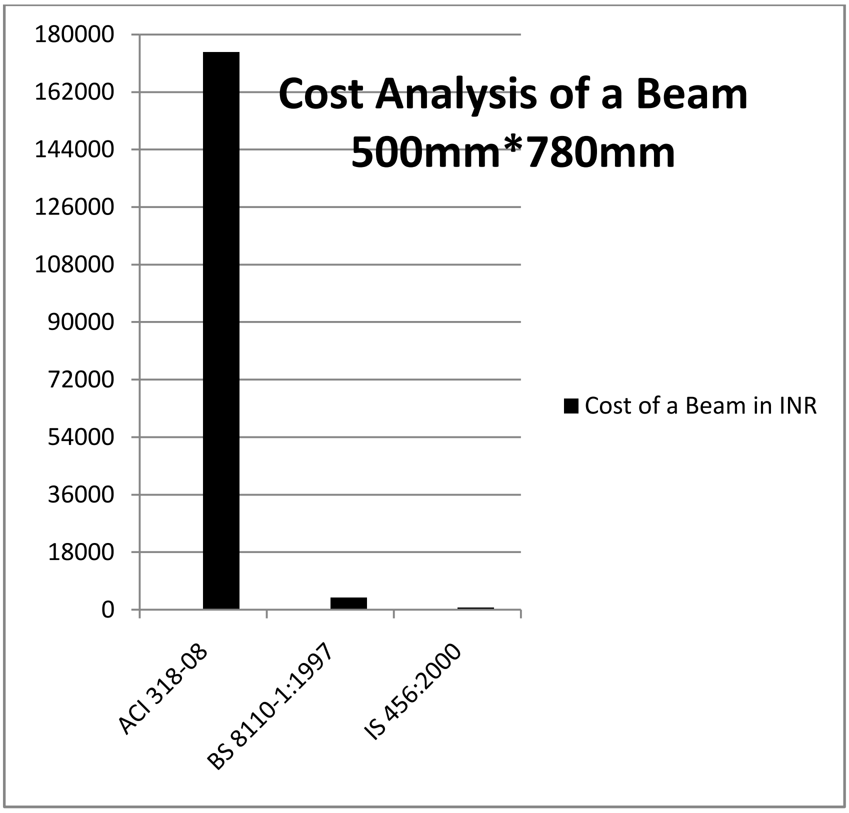
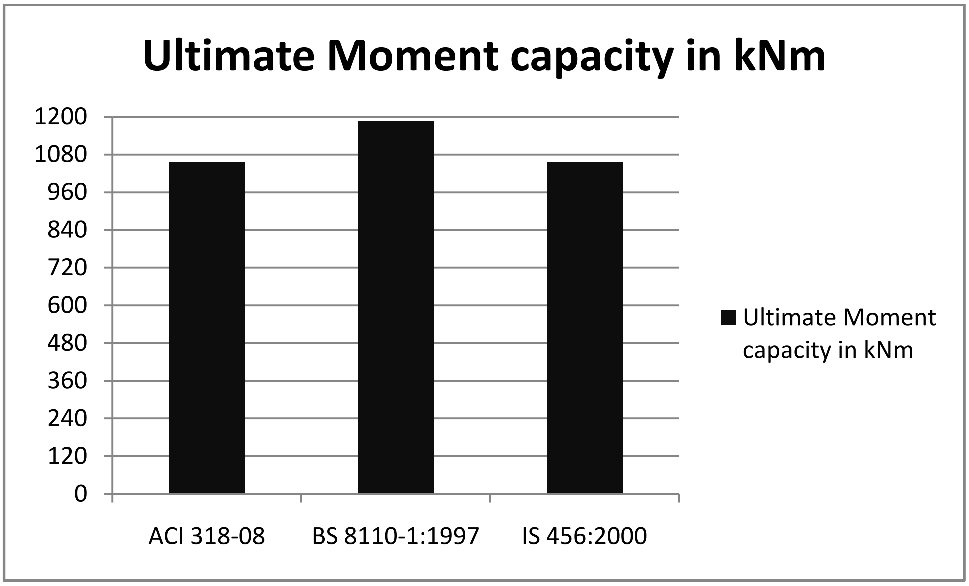


Chart 2: Ultimate Moment of a section 500mm*780mm by ACI 318-08, BS 8110-1:1997 and IS 456:2000



The ACI 318-08 use the nominal yield stresses to calculate the section moment and axial load capacity , then reduce these values by a global capacity reduction factor, whereas British code apply different reduction factors to the concrete and steel yield stress, and use these values to calculate the design ultimate capacities.

All the codes have a greater reduction factor in capacity for sections where concrete crushing controls the design, compared with sections where steel yield in tension controls.

The moment acting or the given section of beam is designed for large moment as the factor for British design is 0.95 whereas for American design and Indian design it is 0.85 and 0.87 respectively.

7. CONCLUSION

This comparative study on the design procedures and cost on all three international codes of practice is brought out to learn about the best design practice that can be economical and serviceable.

The BS 8110-1:1997 stands to the above paragraph as it has a good design practice as well as economical. The ultimate moment capacity of ACI 318-08 is okay but uneconomical when compared to other standards. The IS 456:2000 proves okay.

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