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Probability Distribution Modeling of Extremes Rainfall Series in Makassar City using the L-Moments Method

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ABSTRACT— ***Information on probability distribution of extreme rainfall is very important for planning of water resources and studying related to climatic change. The objective of this study is to identify the best fit probability distribution of extreme rainfall series using L-moments method for three rainfall stations in Makassar city for the period 1985-2014. The results of study show that Generalized Logistic distribution (GLO) is the best fit probability model for the annual maximum rainfall at Maritime Meteorological station of Paotere. Meanwhile, Generalized Pareto distribution (GPA) and Generalized Extreme distribution (GEV) were found as the best fit for Biring Romang station of Panakukkang and BBMKG region IV station of Panaikang, respectively.***

Keywords— extreme rainfall, L-moments, and probability distribution

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1. INTRODUCTION

The Extreme rainfall events can have impact on people's life and their environments for many countries (Feng, et al. 2007, Mayooran & Laheetharan 2014). Extreme rainfall is usually defined as the maximum daily rainfall within each year. One of methods can be used to assess the changes of extreme rainfall that is statistical distributions. Probability distribution models are the useful tools for the frequency analysis of extremes rainfall for the flood mitigation and control (Feng, et al. 2007, Li et al. 2015). However, how to choose an appropriate model in a specific study is still a matter of debate (Li et al. 2015).

Many kinds of probability distributions are available to investigate the extreme rainfall and generally, the extreme rainfall data is non-normally distributed at many regions. For example, Li et al. (2015) found that the Generalized Extreme Value (GEV), Burr, and Weibull distributions provide the best fit probability distribution for both annual and seasonal maximum precipitation in Northwest China. Zin et al. (2009), and Shabri and Ariff (2009) showed that the Generalized Logistic distribution (GLO) performances the best fit for the annual maximum rainfall data in Malaysia. Du et al. (2015) and Xia et al. (2014) used the GEV and Generalized Pareto distributions (GPA) to discuss the historical extreme precipitation frequency and its spatio-temporal variations in China. Meanwhile, the Pearson type III (PE3) was found as the best fit probability model for the annual maximum rainfall in Colombo district (Mayooran & Laheetharan 2014). Other statistical distributions, such as Wakeby and Kappa distributions have also been used to model the summer extreme rainfall in Korea (Park et al. 2001, Park & Jung, 2002).

For non-normal distributions, many researches employed the Anderson-Darling and Kolmogorov-Smirnov tests to fit probability distributions, but these tests are normally not powerful enough to find significant departures from an assumed distribution at an individual site (Buishand 1991). For this reason, von Stoch and Zwiers (1999) recommend using L-moments to estimate the higher statistical moments. The L-moment methods introduced by Hosking and Wallis (1997). The methods possible to obtain reasonable estimates for sample sizes as small as 20 without any assumed distribution. The advantage of this method is that a reasonable fit to the entire cumulative distribution function can be made with just a few parameters and these can be compared when the same distribution function is selected by the fitting procedure (Marx & Kinter 2007). The application of L-Moments to fit probability distribution of extreme rainfall series has been done by several researches (Deka et al. 2009, Eslamian & Feizi 2007, Modarres 2010).

The purpose of this study is to apply L-moments method to evaluate the best fit probability distribution of extreme rainfall series. The Gumbel distribution, the Weibull distribution, the GEV distribution, the GLO distribution, the GPA distribution and the PE3 distribution are considered.

2. STUDY AREA AND DATA

Makassar city is the capital of South Sulawesi and is located between 5º 8' 6"South and 119º 24' 17" East. Makassar's climate is tropical with an average temperature ranging from 26.2º C to 29.3º C as well as the annual average of monthly rainfall is 256.08 mm (Central agency on statistics 2010). In this study, the annual maximum of the average daily rainfall data from three stations are considered for the period 1985-2014. The data are obtained from the Meteorological, Climatological, and Geophysical Agency of Makassar, Indonesia. The selected stations were based not only on the completeness of data, but also on the longest period of data variability. The name and location of rainfall stations are displayed in Table 1 and Fig. 1.

Table 1: Name and geographic coordinates of rainfall stations

Name of stations	Latitude (South)	Longitude (East)
Maritime Meteorological of Paotere (MMP)	05°06′49.5″	119°25′11.5″
Biring Romang of Panaikang (BRP)	05°10′32.7″	119°28′45.5″
BBMKG Region IV of Panaikang (BBMKG)	05°00′56.0″	119°00′08.0″

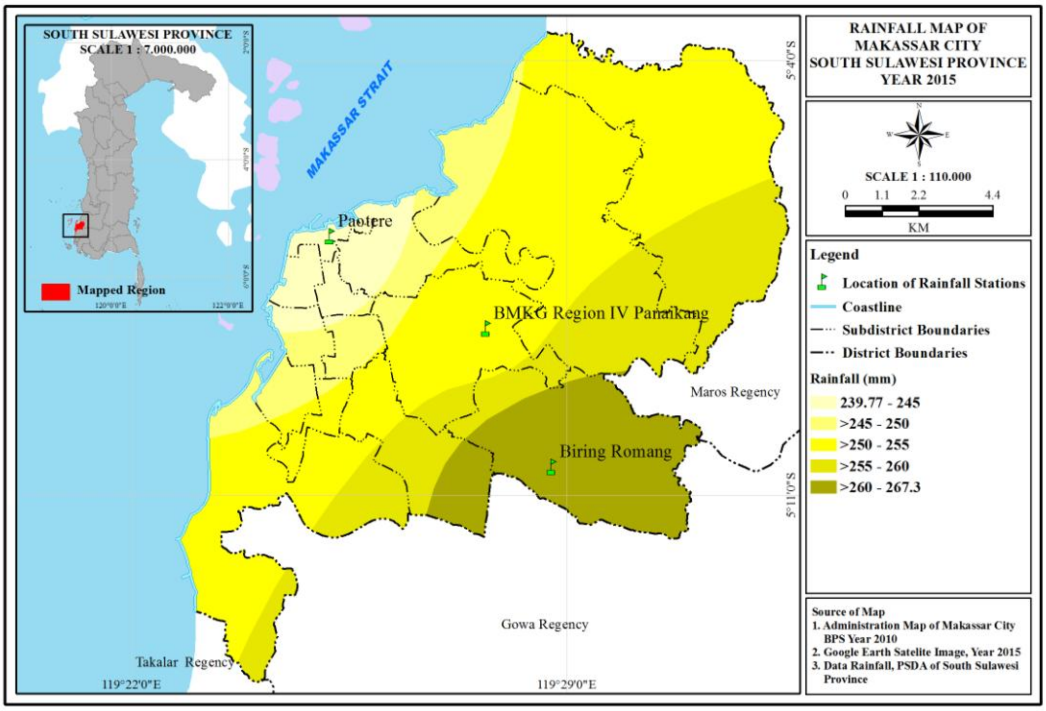

Figure 1: Location of rainfall stations used in this study

3. METHODOLOGY

3.1 Probability Distribution Models

In study, we will fit and compare the performance of five different distributions include Gumbel, GEV, GPD, GLD and PE3 distributions. Advantages of those probability distributions are simple, superior, and popular in frequency analysis of extreme events (Li et al. 2015).

3.1.1 Gumbel distribution

The Gumbel distribution also referred to as the extreme value type I distribution. The distribution can be used to analyze the maximum ranfall data, such as in the flood frequency analysis. It has a cumulative distribution function as follows:

$$F(x) = \exp\left[-\exp\left(-\frac{x-\xi}{\alpha}\right)\right], -\infty < x < \infty, \tag{1}$$

where ξ is location parameter and α is scale parameter $(\alpha > 0)$.

3.1.2 Generalized extreme-value distribution (GEV)

The GEV has a cumulative distribution function as follows:

$$F(x) = \exp[-\exp(-y)], \tag{2}$$

and

$$y = -\kappa^{-1}log\left(1 - \frac{\kappa}{\alpha}(x - \xi)\right), \tag{3}$$

where ξ is location parameter, α is scale parameter, κ is shape parameter and

$$Range\ of\ x: \begin{cases} \left(\xi + \frac{\alpha}{\kappa}\right) < x < \infty, & \kappa < 0 \\ Gumbel\ distribution, & \kappa = 0. \\ -\infty < x < \left(\xi + \frac{\alpha}{\kappa}\right), & \kappa > 0 \end{cases}$$

3.1.3 Generalized Pareto distribution (GPA)

The GPA has a cumulative distribution function as follows:

$$F(x) = 1 - \exp(-y), \tag{4}$$

where

$$y = -\kappa^{-1}log\left(1 - \frac{\kappa}{\alpha}(x - \xi)\right), \tag{5}$$

ξ is location parameter, α is scale parameter, κ is shape parameter and

$$Range\ of\ x: \begin{cases} \left(\xi + \frac{\alpha}{\kappa}\right) \le x < \infty, & \kappa < 0 \\ exponential\ distribution, & \kappa = 0 \\ -\infty < x < \left(\xi + \frac{\alpha}{\kappa}\right), & \kappa > 0 \\ uniform\ distribution, & \kappa = 1 \end{cases}$$

3.1.4 Generalized logistic distribution (GLO)

The GLO has a cumulative distribution function as follows:

$$F(x) = \frac{1}{1 - \exp(y)} \tag{6}$$

where

$$y = -\kappa^{-1}log\left(1 - \frac{\kappa}{\alpha}(x - \xi)\right), \tag{7}$$

ξ is location parameter, α is scale parameter, κ is shape parameter and

$$Range\ of\ x: \begin{cases} \left(\xi + \frac{\alpha}{\kappa}\right) \le x < \infty, & \kappa < 0 \\ logistic\ distribution, & \kappa = 0. \\ -\infty < x < \left(\xi + \frac{\alpha}{\kappa}\right), & \kappa > 0 \end{cases}$$

3.1.5 Pearson Type III Distribution (PE3)

The PE3 has a cumulative distribution function as follows:

$$F(x) = \frac{1}{\alpha\Gamma(\kappa)}\int_0^x \left(\frac{y-\xi}{\alpha}\right)^{\kappa-1} exp\left(-\frac{y-\xi}{\alpha}\right)dy, \tag{8}$$

where $\Gamma(.)$ denotes the Gamma function

$$\Gamma(\kappa) = \int_0^\infty (t)^{\kappa-1} exp(-t)dt,$$

and ξ is location parameter, α is scale parameter, κ is shape parameter.

3.2 L-moments

The L-moments are the summary statistics for probability distributions and data samples and are analogous to ordinary moments (Hosking & Wallis 1997). They provide measures of location, dispersion, skewness, kurtosis, and other aspects of the shape of probability distributions or data samples. L-moment is computed linearly (hence the L) giving a more robust estimate for a given amount of data than other methods (Eslamian & Feizi 2007).

L-moments are linear combinations of probability weighted moments (PWM). Let $x_{1:n} \le \cdots \le x_{n:n}$ be the ordered sample and n is the sample size. Hosking and Wallis (1997) gave an estimator of PWB, β_r, as follows:

$$b_r = \hat{\beta}_r = \frac{1}{n}\sum_{j=r+1}^{n} \frac{(j-1)(j-2)\dots(j-r)}{(n-1)(n-2)\dots(n-r)} x_{j:n}, \qquad r = 1, 2, \dots \qquad (9)$$

The first four L-moments are given by:

$$\lambda_1 = b_0 \qquad (10)$$
$$\lambda_2 = 2b_1 - b_0 \qquad (11)$$
$$\lambda_3 = 6b_2 - 6b_1 + b_0 \qquad (12)$$
$$\lambda_4 = 20b_3 - 30b_2 + 12b_1 - b_0 \qquad (13)$$

where λ_1 is the measure of location (*L*-mean) and λ_2 is the *L*-scale.

Hosking and Wallis (1997) defined the L-moment ratios in hydrological extreme analysis as follows:

$$\tau = \frac{\lambda_2}{\lambda_1}, 0 \le \tau < 1, \qquad (14)$$
$$\tau_3 = \frac{\lambda_3}{\lambda_2}, \qquad (15)$$
$$\tau_4 = \frac{\lambda_4}{\lambda_2}, \qquad (16)$$

where τ is the measure of coefficient of variation (L-C_v), τ_3 is the measure of skewness (L-C_s) and τ_4 is the measure of kurtosis (L-C_k). Unlike standard moments, τ_3 and τ_4 are constrained to be between -1 and $+1$ and τ_4 is further constrained by τ_3 to be no lower than -0.25. Meanwhile, bounds for τ_3 given τ are $(2\tau - 1) \le \tau_3 < 1$ and bounds for τ_4 given τ_3 are $\frac{1}{4}(5\tau_3^2 - 1) \le \tau_4 < 1$. Details about of L-moments method can be found in Hosking and Wallis (1997).

3.3 L-Moment Ratio Diagram

L-Moment ratio diagram is useful tools for visual inspection of candidate distributions and also for goodness-of-fit tests (Modarres 2010, Dodangeh et al. 2011). The L-moments ratio diagram is a plot of L-C_k against L-C_s which can be used to select a suitable probability distribution for a station. A two-parameter distribution is plotted as a single point on the diagram, while a three- parameter distribution as a line. The best fit distribution is obtained, if the (τ_3, τ_4) point lie above the candidate distribution line and distance between (τ_3, τ_4) point and (τ_3, τ_4^{Dist}) point is closest, where τ_4^{Dist} is kurtosis value of candidate distribution and τ_3, τ_4, respectively are skewness and kurtosis values for observed data (Hosking & Wallis 1997). In this study, calculating L-moments values and plotting L-moments ratio are obtained by using software R with *lmom* package.

4. RESULTS AND DISCUSSIONS

According to Fig. 2- Fig. 4, the distribution of monthly extreme rainfall of all the stations for the period 1985-2014. Those figure show that the extreme (maximum) values are evident from December to February, while a dry period occurs from Mei to October (Fig. 3 and Fig. 4), except in Fig. 2, the higher extreme value occurred on June 2002.

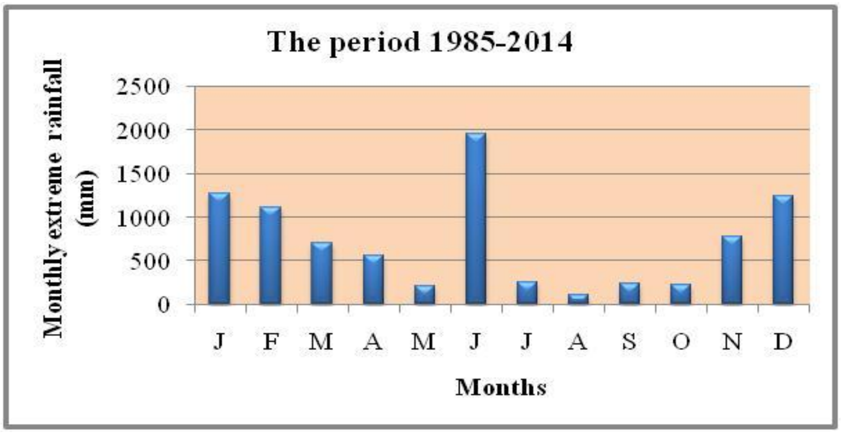

Figure 2: Distribution of monthly extreme rainfall for the MMP station

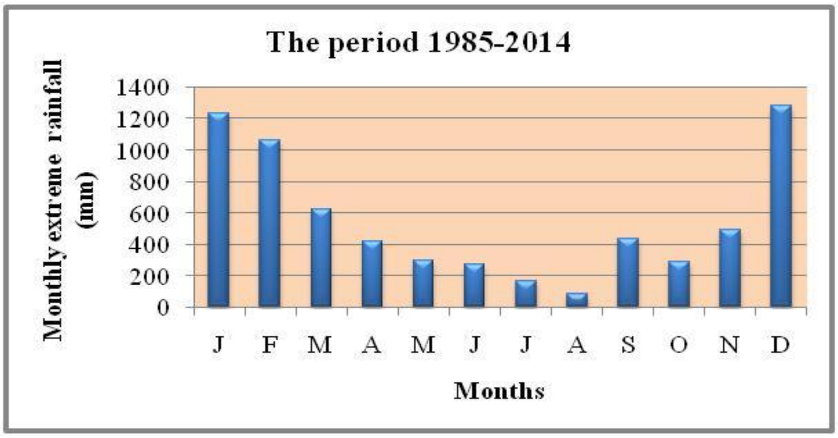

Figure 3: Distribution of monthly extreme rainfall for the BRP station

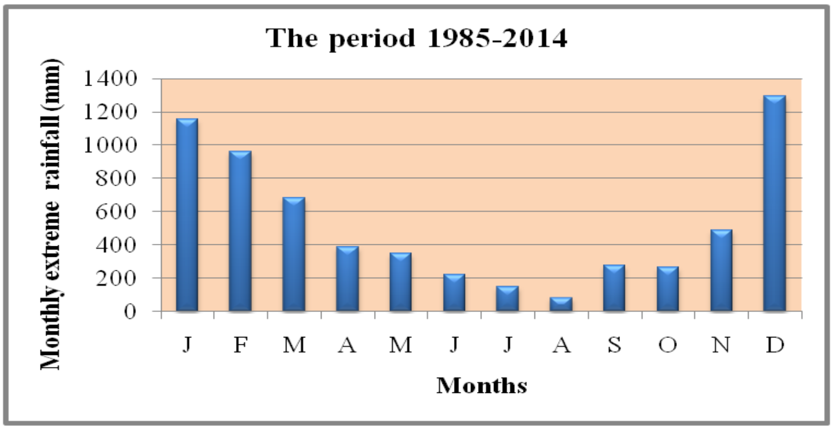

Figure 4: Distribution of monthly extreme rainfall for the BBMKG station

The extreme rainfall can be characterized by mean (λ_1) and coefficient of variation (τ). Table 2 displays those values for all the stations and it can be concluded that the MMP station has the highest mean, while the mean values of the BRP and BBMKG stations are almost same. The variation coefficient value for the BRP station is found to be higher as compared to the other stations. The study result also investigates that the extreme rainfall in this station is relatively more spread as compared to the MMP and BBMKG stations.

Table 2: L-moments and L-moments ratios for each station

Name of stations	λ_1	λ_2	λ_3	λ_4	τ	τ_3	τ_4
MMP	29.709	4.259	1.134	1.483	0.143	0.266	0.348
BRP	28.669	4.404	0.412	0.168	0.154	0.094	0.038
BBMKG	28.612	3.982	0.056	0.447	0.139	0.014	0.112

Table 3: L-kurtosis values (τ_4^{Dist}) for each probability distribution

Name of stations	GUM	GEV	GPA	GLO	PE3
MMP	0.1504	0.1957	0.1179	0.2258	0.1481
BRP	0.1504	0.1251	0.0270	0.1740	0.1252
BBMKG	0.1504	0.1089	0.0030	0.1668	0.1227

In Table 2 also provides L-skewness (τ_3) and L-kurtosis (τ_4) values of all the stations, while Table 3 presents L-kurtosis (τ_4^{Dist}) values for candidate distributions of all the stations. Those values are then plotted to determine the suitable distribution as displayed in Fig 5-Fig 7. A Gumbel distribution (G) is plotted as a single point on the diagram, while the other distributions as a line. Under the L-moments ratio diagram, the MMP station as shown by a black dot lie closest to the GLO distribution (Fig. 5). Based on this figure, the GLO distribution has been selected as the best fit distribution for the MMP station. Fig. 6 shows that the GPA distribution is the best fit probability distribution for the BRP station. In meantime, the GEV distribution gives the best fit to the BBMKG station as we can see in Fig. 7. The estimation of parameters for the best fit probability distribution for all the stations are displayed in Table 4. The estimated values are carried out using the software R.

Table 4: Parameter estimates for the best fit distribution

Name of stations	Distributions	Location	Scale	Shape
MMP	GLO	27.907	3.779	-0.266
BRP	GPA	16.964	19.403	0.658
BBMKG	GEV	26.056	6.937	0.259
BBMKG	GEV	26.056	6.937	0.259

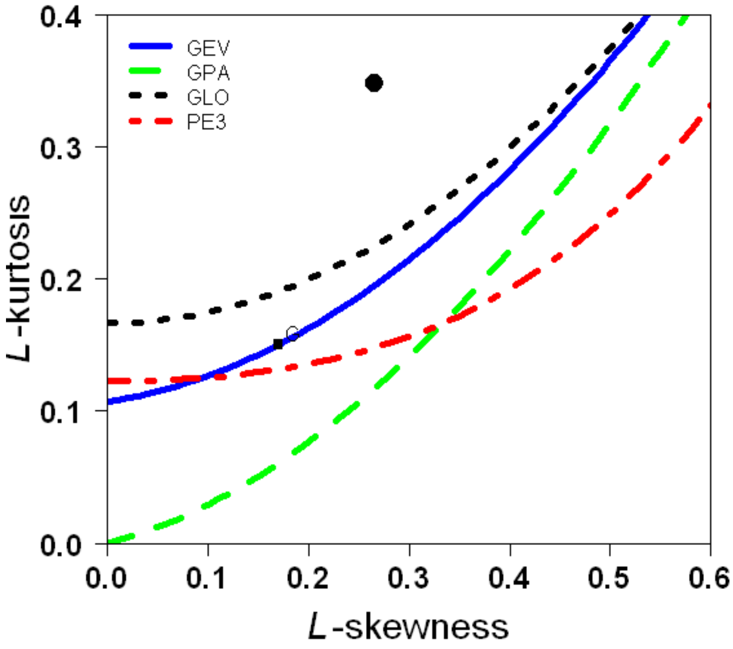

Figure 5: The L-moments ratio diagram for MMP station

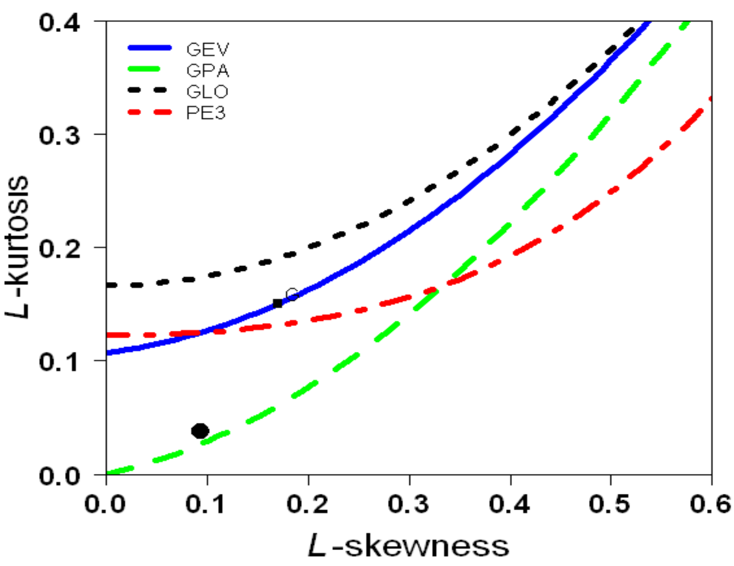

Figure 6: The L-moments ratio diagram for BRP station

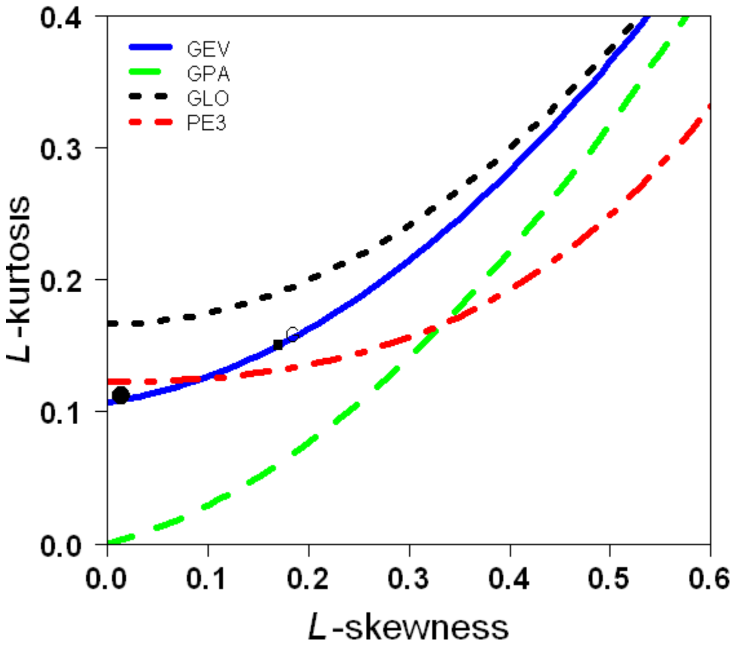

Figure 7: The L-moments ratio diagram for BBMKG station

5. CONCLUSIONS

In this study, the L-moments method has been applied to investigate the performance of five probability distribution for extreme rainfall in Makassar city. The results show that for the MMP station, the GLO distribution is found to be more suitable as compare to the other candidate distributions. The GPA distribution gives the best fit probability distribution for the BRP station, while the GEV distribution is the most suitable distribution to describe extreme rainfall events at the BBMKG station. The study results also agrees with the results obtained by Sanusi et al. (2015). The results from this study can be used as a useful information to estimate the design rainfall in frequency analysis, in particular for extreme rainfall events.

6. ACKNOWLEDGEMENT

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7. REFERENCES

List and number all bibliographical references in 10-point Times New Roman, single-spaced, at the end of your paper. For example, [1] is for a journal paper, [2] is for a book and [3] is for a conference (symposium) paper.

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