Dynamical Studies on the Muon Catalyzed Fusion in the Protium-Tritium Mixture with Very Low Fraction of Tritium

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ABSTRACT— *A general comprehensive network is proposed for computing fusion yield of protium-tritium mixture with low tritium fraction. From this network, the kinetics of the mu-atomic and mu-molecular processes preceding the pt reaction in the $pt\mu$ molecule is described. The time variations of γ quanta and conversion muons and other particles formation in nuclear fusion reactions in $pt\mu$ molecules are studied. Our calculations show that optimum value of muon cycling coefficient at $C_t = 0.01$ is equal to 106. In this paper, our obtained results from theoretical calculations and experimental data are compared with together and we can conclude that the obtained results are in good agreement with measured values.*

Keywords— fusion, protium, tritium, muon

1. INTRODUCTION

Muon catalyzed fusion (μCF) is a physical phenomenon in which negative muon can cause fusion between fusionable particles at room temperature and thereby eliminating the need for high temperature plasmas or powerful lasers. The idea of muon catalyzed fusion (μCF)was first suggested by C.Frank in 1947 ,[1] when he tried to explain tracks from cosmic rays in photoemulsions exposed at high altitudes .Although his explanations was not correct (the tracks were in reality positive muons from pion decays at rest).The experimental discovery of μCF was achieved at the end of 1956 in Berekely by L.W.Alvarez team looking at bubble chamber pictures .[2]A muon (μ) is a leptonic elementary particle and has a finite lifetime of $2.2\mu s$ .Since the mass of a muon 207 times larger than an electron, the size of an exotic atom/molecule containing the negative muon is much smaller than an electronic atom/molecule .When the negative muon binds to hydrogenic nuclei (proton, p, deuteron, d, or triton ,t) like H_2^+ ,a nuclear fusion reactions occurs in the muonic molecular ion, for example:

$$(dt\mu)^+ \rightarrow {}^4He + n + \mu^- \tag{1}$$

The p-t nuclear fusion reaction is one of the reaction that is occurred in the muon catalyzed fusion reaction with a protium-tritium mixture in which we have the least information on all its procedures. For presenting the accuracy of the experimental data and describing the kinetics of all the processes that occur in this mixture we need to have very important information on muon processes in protium-tritium mixture . (e.g., the rate of muon transfer from $p\mu$ atom to triton ,the rate of transition between hyperfine levels of $t\mu$ atoms ,the rate of formation of the $pt\mu$ molecule , the rate of nuclear synthesis in it and muon sticking) From the theoretical point of view, laboratory research on the processes of μCF in hydrogen-tritium mixture, will allow one to represent an algorithm to describe the three body system of interacting particles. Using classical accelerator, essentially study on the pt reaction is impossible in the direct collision at the low energies($\approx$ kev), because the cross sections and proton beam intensities (or tritium) are very small [1-2] .The investigation of the reaction between light nuclei at ultra law energies ($\approx KeV$) is very important for verification of fundamental symmetries in strong interactions[3 − 5],the contribution of muon exchange currents [6 − 9] and to solve some astrophysical problems [10 − 12].There are a few experiments that have examined the properties of μCF in a HT mixture [13-17].In this paper the authors survey the major areas of research : Section II describes the details of the kinetics of the μCF in protium-tritium

mixture, while Sec.III describes the nonlinear point dynamics equations in HT mixture .Sec.IV discusses on the numerical solution of these equations versus time in muon life time range. Sec. V describes the optimum cycling coefficient and energy gain. The final section presents an overview of what is known ,and outline some directions for future research.

2-INTRODUCING OF THE KINETICS OF μCF IN PROTIUM-TRITIUM MIXTURE

Processes of the production of muonic atoms and molecules in a mixture of protium-tritium at very low tritium concentration ($\leq 1\%$) after that negative muons were stopped in this mixture is given in the figure 1. By injecting muon into this mixture, initially $p\mu$ atoms are formed and according to the reaction: $p\mu + t \rightarrow t\mu + p + 183eV$ the muon of $p\mu$ atom is transferred to tritium nucleus. We neglect direct muon capture by tritium , because tritium concentration is very low. $t\mu$ atom, in the ground state is divided into two layers with hyperfine structure of $\vec{F} = \vec{S}_t + \vec{S}_\mu$, where $\vec{F}$ is the total spin of the $t\mu$ atom. ($S_\mu = S_t = \frac{1}{2}$, are respectively muon and triton spin). The initial population of hyperfine layers, statistically are assumed that: $\eta = \frac{1}{4}(F = 0)$ and $\eta = \frac{3}{4}(F = 1)$.By collision of $t\mu$ atoms with H_2 and HT molecules $t\mu + H_2 \rightarrow \left[(pt\mu)^+ pe\right]^+ + e^-$ and $t\mu + HT \rightarrow \left[(pt\mu)^+ te\right]^+ + e^-$ reactions are occurred, respectively. $pt\mu$ molecule, is formed due to electric dipole transition E1 in the excited state (J,υ) (where J and υ ,respectively are rotational -vibrational quantum numbers of pt system inside $pt\mu$ molecule).By collision of $t\mu$ atom with T_2 molecule or HT the formation of $tt\mu$ molecule according the reactions $t\mu + T_2 \rightarrow \left[(tt\mu)^+ te\right]^+ + e^-$ and $t\mu + HT \rightarrow \left[(tt\mu)^+ pe\right]^+ + e^-$ are performed. Also, the transition of $t\mu$ atom between the hyperfine layers according to $t\mu(F = 1) + t \xrightarrow{\lambda_{10}} t\mu(F = 0) + t$ and $t\mu(F = 0) + t \xrightarrow{\lambda_{01}} t\mu(F = 1) + t$ reactions are occurred. Transition is occurred only when the energy of $t\mu$ atom satisfied this condition : $E_{t\mu} \rangle \Delta E = 0.24eV$.(where ΔE is the separated hyperfine energy of ground state of $t\mu$ atom)Transition probability of $t\mu$ atom between the hyperfine layers is due to the reaction of $t\mu(F = 1) + p \underset{\lambda_{01}}{\overset{\lambda_{10}}{\rightleftarrows}} t\mu(F = 0) + p$ in which is very small. [4]Transition of $pt\mu$ molecule from the $(J,\nu) = (1,0)$ state to ground state $(J,\nu) = (0,0)$ occurs very rapidly. $pt\mu$ molecule in the ground state is divided into three separate substrate with the total angular momentum $J = I + S = \frac{3}{2}, \frac{1}{2}, \frac{1}{2}^*$.[4] Complete set of nuclear reactions that are occurred inside the $pt\mu$ molecule and leading to different modes are given in the following:

a) $pt\mu \rightarrow {}^4He + \mu + \gamma + 19.76MeV$

b) $pt\mu \rightarrow {}^4He\mu(1s) + \gamma + 19.77MeV$

c) $pt\mu \rightarrow {}^4He\mu(2p) + \gamma$

d) $pt\mu \rightarrow {}^4He\mu(n \rangle 2) + \gamma$

e) $pt\mu \rightarrow {}^4He\mu(n \rangle 2) + \gamma$

f) $pt\mu \rightarrow {}^4He + \mu + 19.22MeV$

g) $pt\mu \rightarrow {}^4He\mu + e^+ + e^-$

h) $pt\mu \rightarrow {}^4He\mu + \gamma + \gamma$

These reactions are going from the ground state of the ptµ molecule and hence from the s wave of the initial nuclear system [18].Since the total pt spin can be either 1 (initial state 1 $^+$) or 0 (0 $^+$) and the ground state of 4He is 0 $^+$, the possible transitions are 1 $^+ \rightarrow$ 0 $^+$ (M1) and 0 $^+ \rightarrow$ 0 $^+$ (E0).Experimental study of the reactions (a), (b) was made in the only experiment at PSI [19].Events of reaction (g) with creation of electron-positron conversion

pairs have not been observed by the authors. It appeared that the rate of reaction (1) (M1 transition) exceeds the theoretical value eight times and the rate of the muon conversion (E0 transition) is higher than the theoretically expected value by hundreds of times. It is practically impossible to study correctly the pt fusion from the pure s wave in the Beam-target experiments because even for the smallest energies achievable the appropriate transitions (M1 and E0) are masked by the dominant E1 transition from the p wave despite the fact that it is suppressed due to the centrifugal barrier. This is illustrated in Table 1 taken from the latest TUNL measurements [20].

Table 1: Strength of different transitions obtained by the authors of [20] from their investigation of the pt reaction

Collision energy , keV	40	80
E1 strength	% 99.56 ± 0.47	99.8 ± 0.7
M1 strength	% 0.44 ± 0.28	0.2 ± 0.06

Generation of γ quanta with the energy of 19.8MeV (M1 transition) only is possible from $pt\mu$ molecule with total nuclear spin of $I = 1$.Probability of reaction (h), to be negligible small. Numerical values of the decay rate of different channels of $pt\mu$ can be obtained from the following equation:

$$\lambda_{f,i}^{pt}(j) = \rho\left(W_j^0 K_0^i + W_j^1 K_1^i\right) \quad (2)$$

Where $i \equiv \gamma, \mu, e^+, e^-, 2\gamma$ and ρ is the probability density for the case that the distance between proton and triton inside the $pt\mu$ molecule is equal to zero. K_1^i, K_0^i are the pt reaction constants for s-wave in the nuclear cases with $I = 0$ (singlet) and $I = 1$ (triplet) .The ptµ ground state is split into three hyperfine structure (h.f.s.) substates with the total angular momenta j = 3/2, 1/2, 1/2 * [16]. The pt configurations with total spin I have different weights (W) for the ptµ h.f.s. states: state with j = 3/2 corresponds only to I = 1, and both I = 1, 0 can populate the other states. A remarkable feature is that the population of the h.f.s. states of ptµ depends on the tµ-atom spin state F: for F = 1 all three h.f.s. states of ptµ can be occupied and only two states with j = 1/2, 1/2*can be populated for F = 0 (see Table 2). It means that the fusion reaction yields must depend on the tµ-atom spin.

Table2: Population of the ptµ h.f.s levels for different tµ-atom spin states W_j .[21,22]

tµ-atom spin state	j = 3/2	j = 1/2*	j = 1/2
F = 1	0.6667	0.2960	0.0373
F = 0	0	0.8880	0.1120
Statistical mixture F = 1, F = 0	0.5000	0.2500	0.2500

3.DESCRIPTION OF DYNAMICAL EQUATIONS IN THE PROTIUM-TRITIUM MIXTURE WITH VERY LOW TRITIUM CONCENTRATION

With the upsurge of interest in the muon –catalysis of nuclear fusion the need of a comprehensive network for the description of kinetics of associated processes becomes evident.In this section we present as following the point dynamical equations which are the rate equations of various particle by selecting comprehensive network reactions (Fig.1):

$$\frac{dn_\mu}{dt} = S_\mu - \lambda_a C_p \varphi n_\mu + (1-\omega_S)(M\,1_\gamma^{3/2} n_{pt\mu_{3/2}}^{I=1} + M\,1_\gamma^{1/2} n_{pt\mu_{1/2}}^{I=1} + M\,1_\gamma^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} + M\,1_\mu^{3/2} n_{pt\mu_{3/2}}^{I=1}$$

$$+ M\,1_\mu^{1/2} n_{pt\mu_{1/2}}^{I=1} + E\,0_\mu^{1/2} n_{pt\mu_{1/2}}^{I=0} + M\,1_\mu^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} + E\,0_{ee}^{1/2} n_{pt\mu_{1/2}}^{I=0} + M\,1_{ee}^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} + E\,0_\mu^{1/2^*} n_{pt\mu_{1/2^*}}^{I=0}$$

$$+ M\,1_{ee}^{3/2} n_{pt\mu_{3/2}}^{I=1} + M\,1_{ee}^{1/2} n_{pt\mu_{1/2}}^{I=1} + E\,0_{ee}^{1/2^*} n_{pt\mu_{1/2^*}}^{I=0}) \quad (3)$$

$$\frac{dn_{p\mu}}{dt} = \lambda_a c_p \phi n_\mu - W_{t\mu}\eta(F=1)n_{p\mu} - W_{t\mu}\eta(F=0)n_{p\mu} - \lambda_0 n_{p\mu} \qquad (4)$$

$$\frac{dn_{t\mu}^{F=1}}{dt} = W_{t\mu}\eta(F=1)n_{p\mu} + \Lambda_3 n_{t\mu}^{F=0} - \Lambda_2 n_{t\mu}^{F=1} - (k\Lambda_1 + l\lambda_1 + m\Lambda_1 + s\Lambda_1)n_{t\mu}^{F=1} - \lambda_0 n_{t\mu}^{F=1} \qquad (5)$$

$$\frac{dn_{t\mu}^{F=0}}{dt} = W_{t\mu}\eta(F=0)n_{p\mu} + \Lambda_2 n_{t\mu}^{F=1} - \Lambda_3 n_{t\mu}^{F=0} - (n+t+p+u)\Lambda_1 n_{t\mu}^{F=0} - \lambda_0 n_{t\mu}^{F=0} \qquad (6)$$

$$\frac{dn_{pt\mu_{3/2}}^{I=1}}{dt} = k\Lambda_1 n_{t\mu}^{F=1} - M1_\gamma^{3/2} n_{pt\mu_{3/2}}^{I=1} - M1_\mu^{3/2} n_{pt\mu_{3/2}}^{I=1} - M1_{ee}^{3/2} n_{pt\mu_{3/2}}^{I=1} - \lambda_0 n_{pt\mu_{3/2}}^{I=1} \qquad (7)$$

$$\frac{dn_{pt\mu_{1/2}}^{I=1}}{dt} = l\Lambda_1 n_{t\mu}^{F=1} + n\Lambda_1 n_{t\mu}^{F=0} - M1_\gamma^{1/2} n_{pt\mu_{1/2}}^{I=1} - M1_\mu^{1/2} n_{pt\mu_{1/2}}^{I=1} - M1_{ee}^{1/2} n_{pt\mu_{1/2}}^{I=1} - \lambda_0 n_{pt\mu_{1/2}}^{I=1} \qquad (8)$$

$$\frac{dn_{pt\mu_{1/2}}^{I=0}}{dt} = r\Lambda_1 n_{t\mu}^{F=1} + t\Lambda_1 n_{t\mu}^{F=0} - E0_\mu^{1/2} n_{pt\mu_{1/2}}^{I=0} - E0_{ee}^{1/2} n_{pt\mu_{1/2}}^{I=0} - \lambda_0 n_{pt\mu_{1/2}}^{I=0} \qquad (9)$$

$$\frac{dn_{pt\mu_{1/2^*}}^{I=1}}{dt} = m\Lambda_1 n_{t\mu}^{F=1} + p\Lambda_1 n_{t\mu}^{F=0} - M1_\gamma^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} - M1_\mu^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} - M1_{ee}^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} - \lambda_0 n_{pt\mu_{1/2^*}}^{I=1} \qquad (10)$$

$$\frac{dn_{pt\mu_{1/2^*}}^{I=0}}{dt} = s\Lambda_1 n_{t\mu}^{F=1} + u\Lambda_1 n_{t\mu}^{F=0} - E0_{ee}^{1/2^*} n_{pt\mu_{1/2^*}}^{I=0} - E0_\mu^{1/2} n_{pt\mu_{1/2^*}}^{I=0} - \lambda_0 n_{pt\mu_{1/2^*}}^{I=0} \qquad (11)$$

$$\frac{dn_\gamma}{dt} = M1_\gamma^{3/2} n_{pt\mu_{3/2}}^{I=1} + M1_\gamma^{1/2} n_{pt\mu_{1/2}}^{I=1} + M1_\gamma^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} + E0_{ee}^{1/2} n_{pt\mu_{1/2}}^{I=0} + M1_{ee}^{1/2^*} n_{pt\mu_{1/2^*}}^{I=1} +$$

$$E0_\mu^{1/2^*} n_{pt\mu_{1/2^*}}^{I=0} + M1_{ee}^{3/2} n_{pt\mu_{3/2}}^{I=1} + M1_{ee}^{1/2} n_{pt\mu_{1/2}}^{I=1} + E0_{ee}^{1/2} n_{pt\mu_{1/2^*}}^{I=0}) \qquad (12)$$
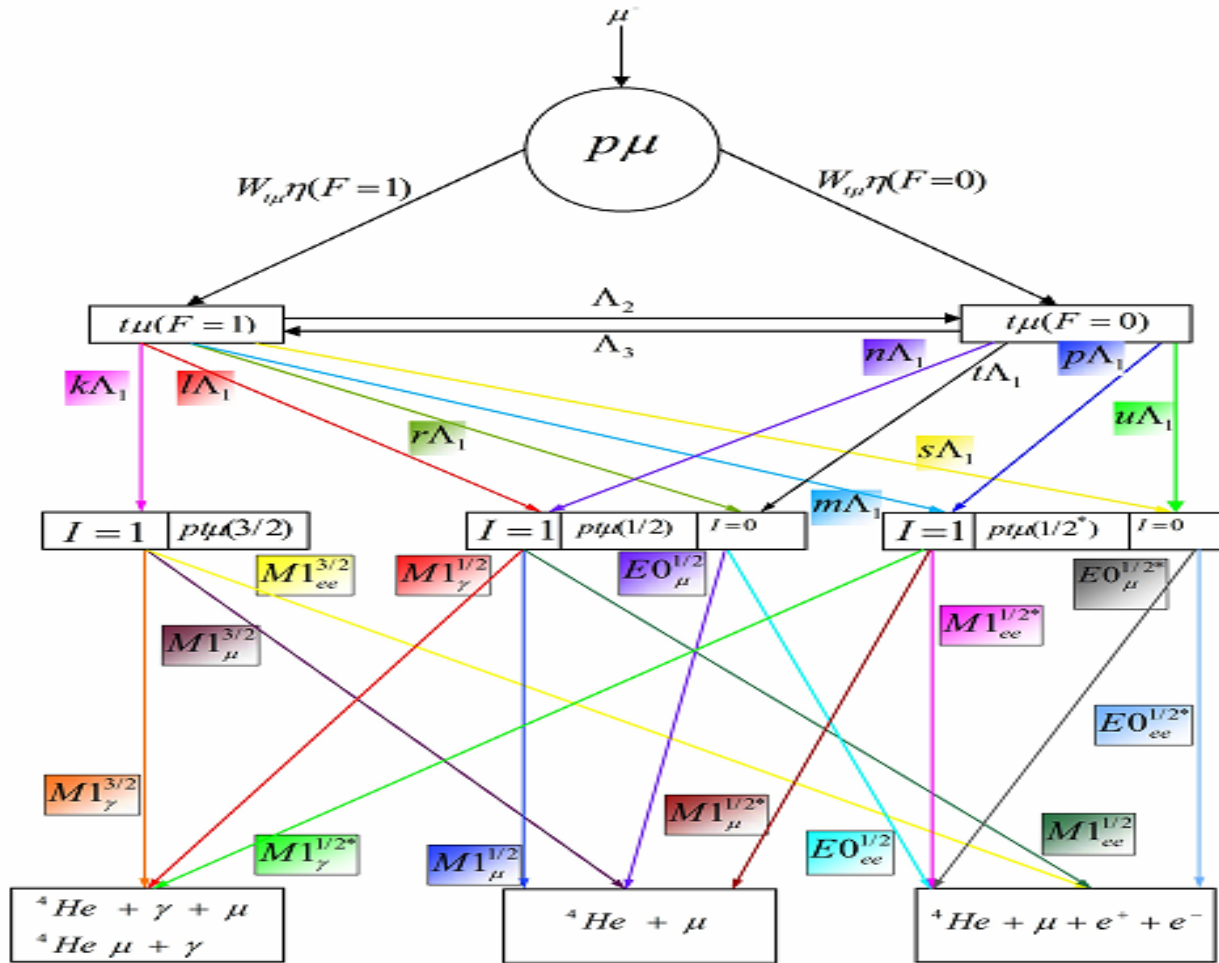
Figure.1: Comprehensive Fusion Reaction Network in Protium-Tritium Mixture with Low tritium Concentration.

where , $n_{(\ )}$ are the particle densities for muons ,atoms ,molecules and γ quanta. $\lambda_0 = 0.445 \times 10^6 \, s^{-1}$ is

the free muon decay rate; $\lambda_{pt}, \lambda_{10}, \lambda_{pt\mu}$ are the rates of the muon transition from $p\mu$ atom to triton ,of the

transition of $t\mu$ atom from the state with $F = 1$ to the state with $F = 0$,and of the $pt\mu$ molecule formation

,respectively (the above values are reduced to liquid hydrogen density

, $n_0 = 4.25 \times 10^{22} \, cm^{-3}$); $\lambda_{f,\mu}^{pt}(I = 0), \lambda_{f,\mu}^{pt}(I = 1)$ are partial rates of nuclear synthesis in the $pt\mu$ molecule

with muon production for the total spin of proton and triton equal to 0

and1,respectively; $\lambda_{f,ee}^{pt}(I = 0), \lambda_{f,ee}^{pt}(I = 1)$ are the rates of nuclear synthesis in the $pt\mu$ molecule with the

formation of an electron-positron pair for the total spin of p and t equal to 0 and 1 ,respectively; C_p and C_t

are atomic concentrations of protium and tritium in protium-tritium mixture ; ϕ is the density of the protium-

tritium mixture reduced to liquid hydrogen density ,also E0 and M1 are the electric dipole and magnetic

dipole transitions rate respectively. In, table.3 , the required numerical values used for solving these

equations and calculation of muon cycling coefficient in protium-tritium mixture are given.

4.MUON STICKING COEFFICIENT

Muon sticking occurs when the muon following fusion becomes bound. The sticking of muons to charged particles after fusion , is an unwanted process and eliminate muons from the chain of fusion reactions. This process is the main loss mechanism in the μCF . After muon catalyzed protium-tritium fusion the muon follows one of three courses: immediate freedom, a short confinement with liberation via subsequent collisions or a life sentence as exhibited by the

Fig.1 .In pt fusion reaction the muon can stick to ^4He by forming a muonic helium ion (μ^4He).In this case ,the muon

stays bound to the ^4He particle and is therefore lost for the foregoing chain of reactions. There is a certain probability (defined by the "reactivation co-efficient" R) that the muon could be shaken off (muon stripping) during the slowing down of the (μ^4He).atom in collisions with nuclei of the traversed matter. The probability ω_s for "final" sticking, after the μ^4He atom came to rest, is defined as: $\omega_s = \omega_{s0}(1-R)$.The sticking probabilities for pt-fusion reaction is equal to $\omega_s = 0.95$.

5.THE FUSION YIELD COMPUTATION

The number of the catalyzed fusions for each muon ,is called the fusion yield OR cycling coefficient.For a certain volume of fusion reactor core, the protium and tritium are injected. Considering a reactor operating period τ during which a general muon injection rate S_μ occurs. Then, total energy supplied and total nuclear energy released as a consequence of injection can be calculated. By use of these calculated parameters, X_c which represents the average number of pt fusion events catalyzed by one muon is determined:

$$X_c = \frac{\int_0^{\tau+\tau_\mu} \lambda_{pt\mu}^f n_{pt\mu}(t)dt}{\int_0^\tau S_\mu(t)dt} \qquad (13)$$

where

$$\lambda_{pt\mu}^f n_{pt\mu}(t) = \lambda_{pt\mu}^{f,I=1}\left(n_{pt\mu\frac{3}{2}}^{I=1}(t) + n_{pt\mu\frac{1}{2}^*}^{I=1}(t)\right) + \lambda_{pt\mu}^{f,I=0}\left(n_{pt\mu\frac{1}{2}}^{I=0}(t) + n_{pt\mu\frac{1}{2}^*}^{I=0}(t)\right) \qquad (14)$$

and $\tau_\mu = 2.2 \times 10^{-6}s$ is the muon life time. To determine X_c , we have to solve the equations (3-12) in time interval $0 \le t \le \tau_\mu$ .Also the energy gain is defined as :

$$G = \frac{Q_{pt}X_c}{E_\mu} \qquad (15)$$

$Q_{pt} = 3.5MeV$ is the fusion energy of the pt- reaction and $E_\mu = 4.5GeV$ is the required energy for production of muon.[23 – 24]

6. NUMERICAL CALCULATIONS

The numerical values of X_c and G are calculated by using Eq.(13) and (15) for various relative concentration ratio of $C_t(0 \le C_t \le 0.01)$,and $\phi = 1LHD$ for the available data given in Table .3.The optimum values of X_c and G with selection of $C_t = 0.01$ are equal to 106 and 0.08, respectively. Therefore, at $C_t = 0.01$, the time variation of numerical values of the different particle densities are given at Table .4. In figs.2 to 7 , time variation of different particle densities such as n_μ , $n_{p\mu}$, $n_{t\mu}^{F=0}$, $n_{t\mu}^{F=1}$, $n_{pt\mu_{3/2}}^{I=1}$, $n_{pt\mu_{1/2}}^{I=0}$, $n_{pt\mu_{1/2}}^{I=1}$, $n_{pt\mu_{1/2}^*}^{I=0}$, $n_{pt\mu_{1/2}^*}^{I=1}$ and n_γ as a function of C_t are shown, respectively.

Table.3: The required numerical values used for calculation of muon cycling coefficient in protium-tritium mixture.[22 – 26]

| PARAMETER	VALUE	PARAMETER	VALUE
$M\,1_\gamma^{3/2} = W_{3/2}^1 \lambda_{f,\gamma(I=1)}^{pt}$	1.57738×10^5	λ_{pt}	$9.3 \times 10^9 s^{-1}$
$M\,1_\mu^{3/2} = W_{3/2}^1 \lambda_{f,\mu(I=1)}^{pt}$	7.8869×10^{-1}	λ_{10}	$6 \times 10^9 s^{-1}$
$M\,1_{ee}^{3/2} = W_{3/2}^1 \lambda_{f,ee(I=1)}^{pt}$	5.40816×10^2	$\lambda_{pt\mu}$	$7.5 \times 10^6 s^{-1}$
$M\,1_\gamma^{1/2} = W_{1/2}^1 \lambda_{f,\gamma(I=1)}^{pt}$	6.76165×10^4	$\lambda_f^{pt}(I=1)$	$6.5 \times 10^4 s^{-1}$
$M\,1_{ee}^{1/2} = W_{1/2}^1 \lambda_{f,ee(I=1)}^{pt}$	2.31828×10^2	$\lambda_f^{pt}(I=0)$	$15 \times 10^4 s^{-1}$

$M\,1^{1/2*}_{\gamma}=W^{1}_{1/2*}\lambda^{pt}_{f,\gamma(I=1)}$	2.3163×10^{3}	$\lambda^{pt}_{f,\gamma}(I=1)$	$0.07\times10^{6}s^{-1}$
$M\,1^{1/2*}_{\mu}=W^{1}_{1/2*}\lambda^{pt}_{f,\mu(I=1)}$	1.15815×10^{-2}	$\lambda^{pt}_{f,ee}(I=1)$	$2.4\times10^{2}s^{-1}$
$M\,1^{1/2*}_{\gamma}=W^{1}_{1/2*}\lambda^{pt}_{f,ee(I=1)}$	7.9416	$\lambda^{pt}_{f,ee}(I=0)$	$3.6\times10^{2}s^{-1}$
$E\,0^{1/2}_{\mu}=W^{0}_{1/2}\lambda^{pt}_{f,\gamma(I=0)}$	1.63905×10^{1}	$\lambda^{pt}_{f,\mu}(I=1)$	$0.35s^{-1}$
$E\,0^{1/2}_{ee}=W^{0}_{1/2}\lambda^{pt}_{f,ee(I=0)}$	1.180116×10^{1}	$\lambda^{pt}_{f,\mu}(I=0)$	$10^{5}s^{-1}$
$E\,0^{1/2*}_{\mu}=W^{0}_{1/2*}\lambda^{pt}_{f,\mu(I=0)}$	4.8275×10^{2}	φ	$1LHD$
$E\,0^{1/2*}_{ee}=W^{0}_{1/2*}\lambda^{pt}_{f,ee(I=0)}$	3.4758×10^{2}	$C_{p}+C_{t}$	$1,C_{t}\le1\%$
$\Lambda_{1}=\phi c_{p}\lambda_{pt\,\mu}$	$6.38\times10^{6}\times C_{p}$	S_{μ}	$1\left(cm^{-3}\right)$
$\Lambda_{2}=\phi c_{t}\lambda_{10}$	$9.1\times10^{8}\times C_{t}$	ω_{s}	0.95
$\Lambda_{3}=\phi c_{t}\lambda_{01}$	$9.1\times10^{8}\times C_{t}$	$n_{\mu}(t=0)$	$1,cm^{-3}$
k	6.67×10^{-1}	$n_{p\mu}(t=0)$	$0,cm^{-3}$
l	3.60×10^{-2}	$n^{F=1}_{t\mu}(t=0)$	$0,cm^{-3}$
m	9.80×10^{-3}	$n^{F=0}_{t\mu}(t=0)$	$0,cm^{-3}$
n	8.59×10^{-1}	$n^{I=0}_{pt\,\mu_{3/2}}(t=0)$	$0,cm^{-3}$
p	3.70×10^{-3}	$n^{I=1}_{pt\,\mu_{1/2}}(t=0)$	$0,cm^{-3}$
r	1.22×10^{-3}	$n^{I=0}_{pt\,\mu_{1/2*}}(t=0)$	$0,cm^{-3}$
s	0.286	$n^{I=1}_{pt\,\mu_{1/2*}}(t=0)$	$0,cm^{-3}$
t	2.93×10^{-2}	$n^{I=1}_{pt\,\mu_{3/2}}(t=0)$	$0,cm^{-3}$
u	0.108	$n_{\gamma}(t=0)$	$0,cm^{-3}$

Table.4: The time variation of numerical values of the different particle densities in protium-tritium mixture at $C_{t}=0.01$

t	10^{-15}	10^{-12}	10^{-9}	10^{-8}	10^{-7}	10^{-6}	2.2×10^{-6}
n_{μ}	0.99604	0.01906	1.9904×10^{-11}	2.1879×10^{-10}	1.5951×10^{-9}	1.5948×10^{-9}	5.7191×10^{-10}
$n_{p\mu}$	0.00395	0.97568	0.00091	1.2218×10^{-7}	9.0150×10^{-7}	9.0227×10^{-7}	3.2355×10^{-7}
$n^{F=0}_{t\mu}$	3.4604×10^{-9}	0.00131	0.24836	0.23441	0.13031	0.00063	0.00009
$n^{F=1}_{t\mu}$	1.0381×10^{-8}	0.00393	$.74492$	0.70155	0.38172	0.00165	0.00028
$n^{I=1}_{pt\,\mu_{3/2}}$	1.4437×10^{-17}	6.7750×10^{-9}	0.00267	0.02973	0.21950	0.27718	0.13472
$n^{I=0}_{pt\,\mu_{1/2}}$	2.3781×10^{-19}	1.1159×10^{-10}	0.00004	0.00049	0.00368	0.00538	0.00315
$n^{I=1}_{pt\,\mu_{1/2}}$	6.9771×10^{-18}	3.2740×10^{-9}	0.00129	0.01434	0.10411	0.09044	0.02397
$n^{I=0}_{pt\,\mu_{1/2*}}$	6.9699×10^{-18}	3.2706×10^{-9}	0.00130	0.01439	0.10965	0.22282	0.22227
$n^{I=1}_{pt\,\mu_{1/2*}}$	2.3882	1.1207	0.00004	0.0004	0.00363	0.00460	0.00223

$\times 10^{-19}$	$\times 10^{-10}$					
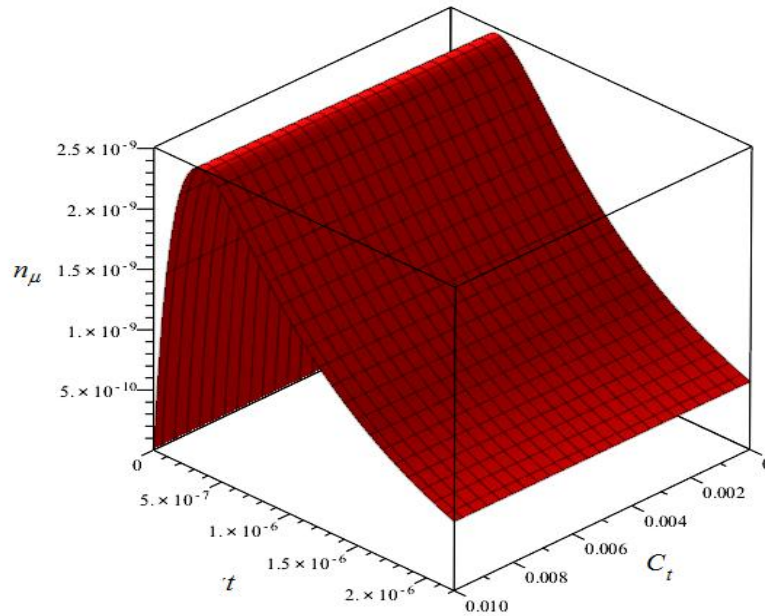
Figure.2: Variation of n_{μ} at Time Interval $0\langle t \langle 2.2 \times 10^{-6}\, s$ as a Function of C_t.
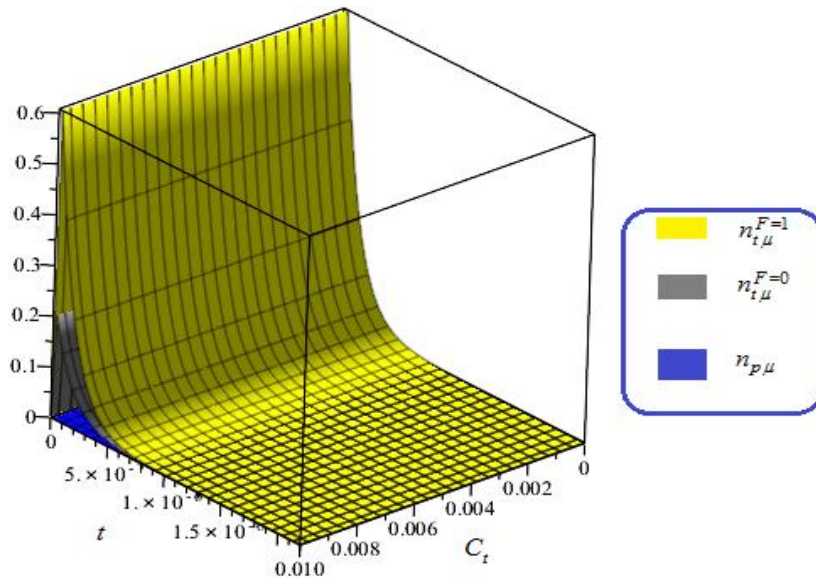
Figure.3: Variations of $n_{p\mu}$, $n^{F=0}_{t\mu}$ and $n^{F=1}_{t\mu}$ at Time Interval $0\langle t \langle 2.2 \times 10^{-6}\, s$ as a Function of C_t.
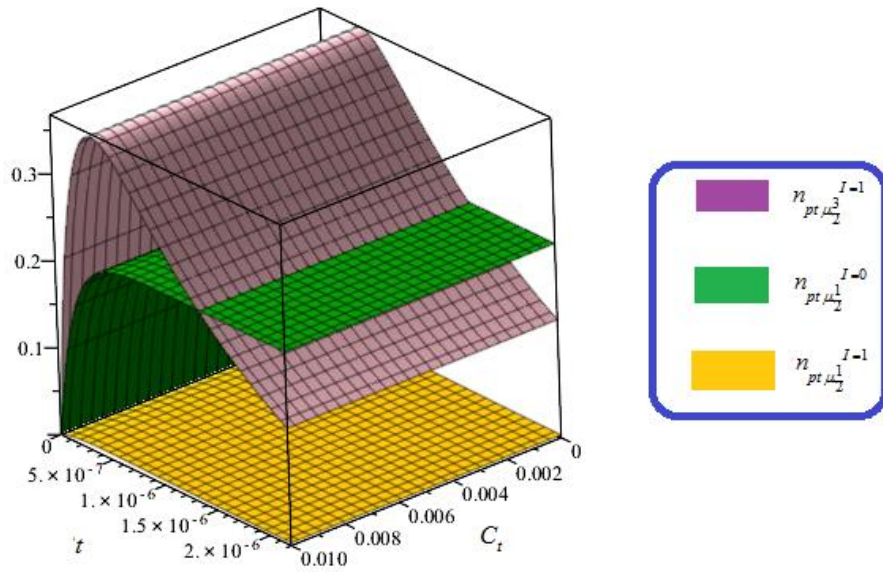
Figure.4: Variations of $n^{I=1}_{pt\,\mu\frac{3}{2}}$, $n^{I=1}_{pt\,\mu\frac{1}{2}}$ and $n^{I=0}_{pt\,\mu\frac{1}{2}}$ at Time Interval $0\langle t\langle 2.2\times10^{-6}\,s$ as a Function of C_t.
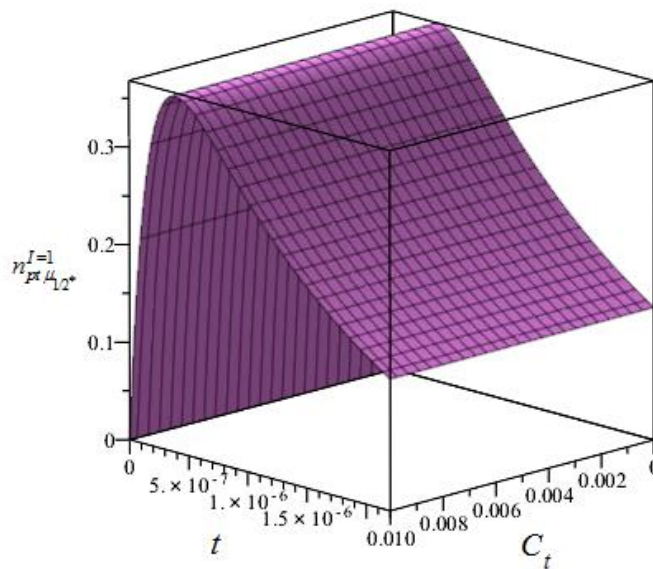
Figure.5: Variations of $n^{I=1}_{pt\,\mu\frac{1}{2}^*}$ at Time Interval $0\langle t\langle 2.2\times10^{-6}\,s$ as a Function of C_t.
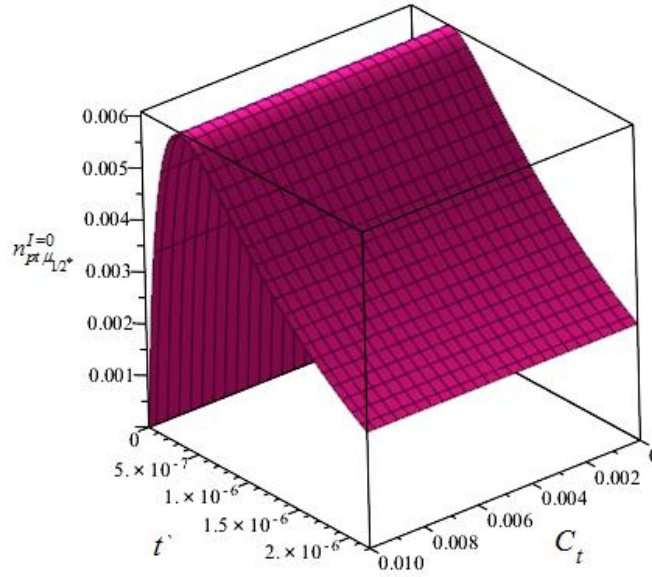
Figure.6: Variations of $n^{I=0}_{pt\mu\frac{1}{2}^*}$ at Time Interval $0\langle t \langle 2.2 \times 10^{-6}\,s$ as a Function of C_t
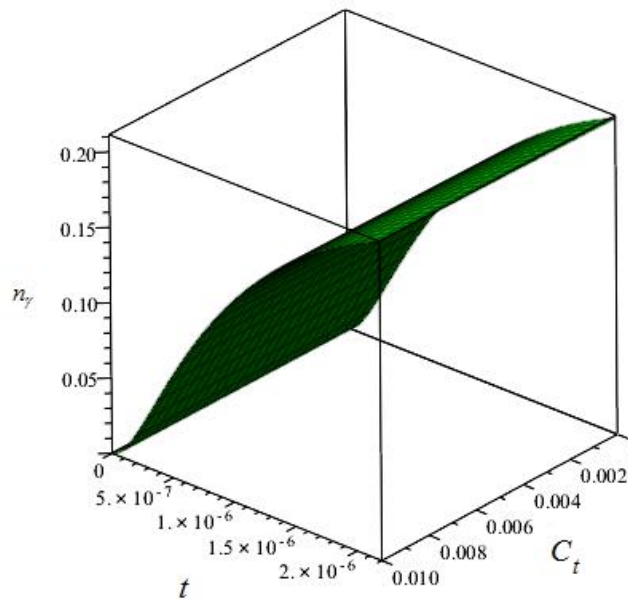
Figure.7: Variations of n_γ at Time Interval $0\langle t \langle 2.2 \times 10^{-6}\,s$ as a Function of C_t

From observing figs.2 to 7 we can find that the density of n_μ , $n_{p\mu}$, $n^{F=0}_{t\mu}$, $n^{F=1}_{t\mu}$, $n^{I=1}_{pt\mu\frac{3}{2}}, n^{I=1}_{pt\mu\frac{1}{2}^*}, n^{I=0}_{pt\mu\frac{1}{2}^*}$ initially by increasing time are increased ,and then by increasing time are decreased. but density of $n_{p\mu}$ is very small in comparing to $n^{F=0}_{t\mu}$ and $n^{F=1}_{t\mu}$ .Also, from fig.4,we see that ,the time variations of $n^{I=1}_{pt\mu\frac{1}{2}}$ is smaller than $n^{I=1}_{pt\mu\frac{3}{2}}$ and $n^{I=0}_{pt\mu\frac{1}{2}}$ .Finally from fig.7 we see that by increasing time the value of n_γ is increased.

7.CONCLUSION AND DISCUSSION

The "energy cost " for the creation of a muon is one of the most important practical parameters in analysing the relevance of muon catalysed fusion for energy production. The muons are produced during the decay of pions which can

be created in nucleon-nucleon collisions. The energy threshold for pion generation is about 500 MeV of projectile kinetic energy and the process of negative muon generation seems to be most effective for nucleonic projectiles with a kinetic energy of 1 GeV per nucleon. An optimistic estimate requires 5 GeV of energy to produce one negative muon.[30].Therefore if one muon catalyses about one hundred dt fusions the energy output is less than 2 Gev (one dt fusion gives 17.6 MeV)per muon ,so that no energy gain seems to be possible from this fusion nuclear reaction. Thus, instead of this mixture, we select protium-tritium mixture with low tritium fraction. From the presented analysis of the μCF kinetic in the protium-tritium mixture we can conclude that the muon cycling coefficient in $pt\mu$ ($X_c = 106, \phi = 1LHD, C_t = 0.01, T = 300K$) is increasing respect to muon cycling coefficient of $pt\mu$ branch in protium-tritium-deuterium   mixture($X_c = 10, \phi = 1LHD, C_t = C_d = 0.49, T = 300K$),[24] this is due to that in the protium-tritium-deuterium mixture, the other fusion reaction branches($dt\mu, dd\mu, pd\mu, tt\mu$)are formed, and $dt\mu$ branch has maximum value muon cycling coefficient approximately equal to 111 because the $dt\mu$ molecule has a maximum fusion rate and minimum muon sticking coefficient,$[25]$ but one of the advantage of the selection of protium-tritium mixture respect to the protium-tritium-deuterium mixture is that ,for reaching to maximum value cycling rate in protium-tritium mixture   only we required to tritium concentration C_t=0.01, but in the protium-tritium-deuterium mixture tritium concentration is increased . The other advantages of protium-tritium mixture is imply on tritium , is radioactive and its production is very expensive. The disadvantage of protium-tritium mixture is muon sticking coefficient in $pt\mu$ molecule is very high. Therefore, for future research, we must study on the reduction muon sticking in protium-tritium mixture. Also we can deduce that, the TRIMUF measurements confirm our predicted theory.[27,28,29]

8. ACKNOWLEDGEMENT

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