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On Soft $I_{\pi g}$ - Normality and Soft $I_{\pi g}$ - Regularity

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ABSTRACT---- ***The concept of soft sets was introduced by Molodtsov[12] as a general mathematical tool for dealing with uncertain objects. He successfully applied the soft set theory in several directions such as smoothness of functions, game theory, operations research, Riemann integration, Perror integration, probability, theory of measurement and so on. Shabir and Naz[16] appilied this theory to topological structure and studied the concept of soft topological spaces. A soft ideal on a non empty set X is a non empty collection of soft subsets with heredity property which is closed under finite unions. The notion of soft ideal was first given by R. Sahin and A. Kucuk[14].In this paper we focus our study on the concept of soft $I_{\pi g}$ - normality and soft $I_{\pi g}$ - regularity by the definition of soft ideal and various characterizations and properties are given. Furthermore we present the behaviors and features of soft mildly normal spaces and soft almost regular spaces.***

Keywords---- soft $I_{\pi g}$ - open set, soft $I_{\pi g}$ - closed set, soft $I_{\pi g}$ - normal space, soft $I_{\pi g}$ - regular space, soft mildly normal space, soft almost regular space.

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1. INTRODUCTION

Soft set theory proposed by Molodtsov[12] has been regarded as an effective mathematical tool to deal with uncertainty, which associates a set with a set of parameters. Muhammad Shabir and Munazza Naz[16] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. The topic of ideals in general topological spaces has an excellent potential for applications in other branch of mathematics. It was the work of Jankovic and Hamlett [8] which motivated the research in applying topological ideals to generalize the most basic properties in soft topology. The notion of soft ideal was first given by R. Sahin and A. Kucuk[14]. A soft ideal on a nonempty set X is a nonempty collection of soft subsets of X with heredity property which is also closed under finite union. Then Mustafa and Sleim[13] defined a different version of soft ideal. By the light of this definition Kandil et al[10] initiated the concept of soft $*$-topology which was finer than soft topology. Aysegul Caksu Guler and Goknur Kale[4] formulated the notion of soft I-regular spaces and soft I- normal spaces. This paper aims to explore the concept of soft $I_{\pi g}$ - normal space and soft $I_{\pi g}$ - regular space and several characterizations of these concepts are discussed with illustrative examples.

2. PRELIMINARIES

Throughout this paper, X will be a nonempty initial universal set and E will be a set of parameters. Let P(X) denote the power set of X and S(X) denote the set of all soft sets over X.

Definition: 2.1[12]

Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a non- empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by F: A → P(X).

Definition: 2.2[1]

A subset (A, E) of a topological space X is called soft regular closed, if cl(int(A,E)) = (A,E). The complement of soft regular closed set is soft regular open set.

Definition: 2.3[2]

The finite union of soft regular open sets is said to be soft π-open. The complement of soft π-open is said to be soft π-closed.

Definition: 2.4[2]
A subset (A, E) of a topological space X is called soft πg-closed in a soft topological space (X,τ, E), if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X. The complement of soft πg-closed set is soft πg-open set.

Definition: 2.5[16]
Let (X, τ, E) be a soft topological space over X and $x, y \in$ X such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \in$ (F, E) and $y \notin$ (F, E) and $y \in$ (G, E) and $x \notin$ (G, E), then (X, τ, E) is called a soft T_1-space.

Definition: 2.6[16]
Let (X, τ, E) be a soft topological space over X, (G, E) be a soft closed set in X and $x \in$ X such that $x \notin$ (G, E). If there exist soft open sets (F_1, E) and (F_2, E) such that $x \in$ (F_1, E), (G, E) $\subseteq$ (F_2, E) and (F_1, E) $\cap$ (F_2, E) = ϕ, then (X, τ, E) is called a soft regular space.

Definition: 2.7[16]
Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) soft closed sets over X such that (F, E) $\cap$ (G, E) = ϕ. If there exist soft open sets (F_1, E) and (F_2, E) such that (F, E) $\subseteq$ (F_1, E), (G, E) $\subseteq$ (F_2, E) and (F_1, E) $\cap$ (F_2, E) = ϕ, then (X, τ, E) is called a soft normal space.

Definition: 2.8[10]
Let I be a non-null collection of soft sets over a universe X with the same set of parameters E. Then I $\in SS(X)_E$ is called a soft ideal on X with the same set E, if
(1) (F, E) $\in$ I and (G, E) $\subseteq$ (F, E) implies (G, E) $\in$ I

(2) (F, E) $\in$ I and (G, E) $\in$ I implies (F, E) $\cup$ (G, E) $\in$ I

Definition: 2.9[15]
A subset (A, E) of a soft ideal space (X, τ, E, I) is said to be soft $I_{\pi g}$ - closed, if $(A, E)^*$ $\subseteq$(U, E) whenever (A, E) $\subseteq$ (U, E) and (U, E) is soft π-open. The complement of soft $I_{\pi g}$ - closed set is soft $I_{\pi g}$ - open set.

Theorem: 2.10[15]

A subset (A, E) of a soft ideal space (X, τ, E, I) is soft $I_{\pi g}$ - open if and only if (F, E) $\subseteq$ int^*(A, E) whenever (F, E) is soft π- closed and (F, E) $\subseteq$ (A, E).

3. SOFT $I_{\pi g}$ - NORMAL SPACES

Definition: 3.1

A soft ideal space (X, τ, E, I) is said to be a soft $I_{\pi g}$ - normal space, if for every pair of disjoint soft closed sets (A, E) and (B, E), there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (V, E) such that (A, E) $\subseteq$ (U, E) and (B, E) $\subseteq$ (V, E).

Preposition: 3.2

Every soft normal space is soft $I_{\pi g}$ - normal space. But the converse is not true.

Proof: It follows from the fact that every soft open set is soft $I_{\pi g}$ - open set.

Example: 3.3

X = {a, b, c} and E = {e_1, e_2}.

(A, E) = {(e_1, {b}), (e_2, {a})}

(B, E) = {(e_1, {b, c}), (e_2, {a, b})}

(C, E) = {(e_1, {a, b}), (e_2, {a, c})} where (A, E), (B, E),(C, E) soft sets over X and

τ = {$\tilde{X}$, $\tilde{\phi}$, (A, E), (B, E), (C, E)} is a soft topology over X.

Let I = {$\tilde{\phi}$, (I_1, E), (I_2, E), (I_3, E)} be a soft ideal over X, where

(I_1, E) = {(e_1, {a}), (e_2, ϕ)}

(I_2, E) = {(e_1, ϕ), (e_2, {c})}

(I_3, E) = {(e_1, {a}), (e_2, {c})}.

Let (F, E) = {(e_1, {c}), (e_2, {b})} and (G, E) = {(e_1, {a}), (e_2, {c})} be disjoint soft closed sets in X. There exist disjoint soft $I_{\pi g}$ - open sets (U, E) = {(e_1, {b, c}), (e_2, {a, b})} and (V, E) = {(e_1,{a}), (e_2, { c})} such that (F, E) $\subseteq$ (U, E) and (G, E) $\subseteq$ (V, E). Hence X is soft $I_{\pi g}$ - normal space but not soft normal, because (V, E) is not soft open set in X.

Theorem: 3.4

Let (X, τ, E, I) be a soft ideal space. Then the following are equivalent:

(1) X is soft soft $I_{\pi g}$ - normal

(2) For every pair of disjoint soft closed sets (A, E) and (B, E) there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

(3) For every soft closed set (A, E) and a soft open set (V, E) containing (A, E) there exists a soft $I_{\pi g}$ - open set (U, E) such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

Proof:

(1) ⟹ (2)

The proof follows from the definition of soft $I_{\pi g}$ - normal space.

(2) ⟹ (3)

Let (A, E) be a soft closed set and (V, E) be a soft open set containing (A, E). Since (A, E) and

X − (V, E) are disjoint soft closed sets, there exists disjoint soft $I_{\pi g}$ - open sest (U, E) and (W, E) such that (A, E) ⊆ (U, E) and X − (V, E) ⊆ (W, E). Again (U ∩ W, E) = ϕ implies that (U, E) ∩ int^*(W, E) = ϕ. Then cl^*(U, E) ⊆ X − int^*(W, E). Since X − (V, E) is soft closed and (W, E) is soft $I_{\pi g}$ - open, X − (V, E) ⊆ (W, E) implies that X − (V, E) ⊆ int^*(W, E) and so X −int^*(W, E) ⊆ (V, E). Thus (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ X − int^*(W, E) ⊆ (V, E).

(3) ⟹(1)

Let (A, E) and (B, E) be two disjoint soft closed subsets of X. By hypothesis there exists a soft $I_{\pi g}$ - open set (U, E) such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ X − (B, E). If (W, E) = X − cl^*(U, E) then (U, E) and (W, E) are the required disjoint soft $I_{\pi g}$ - open sets containing (A, E) and (B, E) respectively. Hence (X, τ, E, I) is soft $I_{\pi g}$ – normal space.

Theorem: 3.5

Let (X, τ, E, I) be a soft ideal space which is soft $I_{\pi g}$-normal. Then for every soft closed set (A, E) and every soft πg-open set (B, E) containing (A, E), there exist soft $I_{\pi g}$- open set (U, E) such that (A, E) ⊆ int^*(U, E) ⊆ (U, E) ⊆ (B, E).

Proof:

Let (A, E) be a soft closed set and (B, E) be a soft πg-open set containing (A, E). Then (A, E) ∩ (X − (B, E)) = ϕ where (A, E) is soft closed set and X − (B, E) is soft πg-closed set. By theorem: 3.7, there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and X − (B, E) ⊆ (V, E). Since (U∩V, E) = ϕ we have (U, E) ⊆ X − (V, E). Then (A, E) ⊆ int^*(U, E). Hence (A, E) ⊆ int^*(U, E) ⊆ (U, E) ⊆ X − (B, E) ⊆ (B, E).

Theorem: 3.6

Let (X, τ, E, I) be a soft ideal space which is soft $I_{\pi g}$-normal. Then for every soft πg-closed set (A, E) and every soft open set (B, E) containing (A, E), there exist soft $I_{\pi g}$- closed set (U, E) such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ (B, E).

Proof:

Let (A, E) be a soft πg-closed set and (B, E) be a soft open set containing (A, E). Then X − (B, E) is soft π-closed set contained in soft πg-open set X − (A, E). By theorem: 3.10, there exists soft $I_{\pi g}$ - open set (V, E) such that X − (B, E) ⊆ int^*(V, E) ⊆ (V, E) ⊆ X − (A, E). Therefore (A, E) ⊆ X − (V, E) ⊆ cl^*(X − (V, E)) ⊆ (B, E). If (U, E) = X − (V, E) then (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ (B, E). Hence (U, E) is the required soft $I_{\pi g}$- closed set.

Definition: 3.7

A soft ideal space (X, τ, E, I) is said to be a soft $I^*_{\pi g}$ - normal space, if for every pair of disjoint soft $I_{\pi g}$ -closed sets (A, E) and (B, E), there exist disjoint soft open sets (U, E) and (V, E) in X such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

Preposition: 3.8

Every soft $I^*_{\pi g}$ - normal space is soft normal space. But the converse is not true.

Proof:

Since every soft closed set is soft $I_{\pi g}$-closed set, every soft $I^*_{\pi g}$ - normal space is soft normal space.

Example: 3.9

X = {a, b, c, d} and E = {e_1, e_2}.

(A, E) = {(e_1, {a}), (e_2, {d})}

(B, E) = {(e_1, {b}), (e_2, {c})}

(C, E) = {(e_1, {a, b}), (e_2, {c, d})}

(D, E) = {(e_1, {b, c, d}), (e_2, {a, b, c})} where (A, E), (B, E), (C, E) and (D, E) soft sets over X and $\tau = \{\tilde{X}, \tilde{\phi}$, (A, E), (B, E), (C, E), (D, E)} is a soft topology over X. Let I = {$\tilde{\phi}$, (I_1, E)} be a soft ideal over X, where (I_1, E) = {(e_1, {b}), (e_2, {a})}. Let (F, E) = {(e_1, {a}), (e_2, {d})} and (G, E) = {(e_1, {b}), (e_2, {c})} be disjoint soft closed sets in X. Then there exist disjoint soft open sets (U, E) = {(e_1, {a}), (e_2, {d})} and (V, E) = {(e_1, {b}), (e_2, {c})} such that (F, E) ⊆ (U, E) and (G, E) ⊆ (V, E). Hence X is soft normal space but not soft $I^*\pi g$ - normal, because (V, E) is not soft $I_{\pi g}$ -closed set in X.

Theorem: 3.10

In a soft ideal space (X, τ, E, I) the following are equivalent:

(1) X is soft $I^*_{\pi g}$ - normal space

(2) For every soft $I_{\pi g}$-closed set (A, E) and every soft $I_{\pi g}$-open set (B, E) containing (A, E) there exists a soft open set (U, E) of X such that (A, E) ⊆ (U, E) ⊆ cl (U, E) ⊆ (B, E).

Proof:

(1) ⟹ (2)

Let (A, E) be a soft $I_{\pi g}$-closed set and (B, E) be a soft $I_{\pi g}$-open set containing (A, E). Since (A, E) and X − (B, E) are disjoint soft soft $I_{\pi g}$-closed sets, there exists disjoint soft open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and X − (B, E) ⊆ (V, E). Now (U ∩ V, E) = ϕ implies that cl(U, E) ⊆ X − (V, E). Therefore (A, E) ⊆ (U, E) ⊆ cl (U, E) ⊆ X − (V, E) ⊆ (B, E).

(2) ⟹ (1)

Suppose (A, E) and (B, E) are disjoint soft $I_{\pi g}$-closed sets, then the soft $I_{\pi g}$-closed set (A, E) is contained in the soft soft $I_{\pi g}$-open set X − (B, E). By hypothesis there exists a soft open set (U, E) of X such that (A, E) ⊆ (U, E) ⊆ cl (U, E) ⊆ X − (B, E). If (V, E) = X − cl(B, E), then (U, E) and (V, E) are disjoint soft open sets containing (A, E) and (B, E) respectively. Therefore X is soft $I^*_{\pi g}$ - normal space.

Theorem: 3.11

In a soft ideal space (X, τ, E, I) the following are equivalent:

(1) X is soft $I^*_{\pi g}$ - normal space

(2) For each pair of disjoint soft $I_{\pi g}$-closed subsets (A, E) and (B, E) of X there exists a soft open set (U, E) of X containing (A, E) such that cl(U, E) ∩ (B, E) = ϕ.

(3) For each pair of disjoint soft $I_{\pi g}$-closed subsets (A, E) and (B, E) of X there exists a soft open set (U, E) of X containing (A, E) and a soft open set (V, E) containing (B, E) such that cl(U, E) ∩ cl(V, E) = ϕ.

Proof:

(1) ⟹ (2)

Suppose that (A, E) and (B, E) are disjoint soft $I_{\pi g}$-closed subsets of X. Then the soft $I_{\pi g}$-closed set (A, E) is contained in the soft $I_{\pi g}$- open set X − (B, E). Then there exists a soft open set (U, E) such that (A, E) ⊆ (U, E) ⊆ cl (U, E) ⊆ X − (B, E). Therefore (U, E) is the required soft open set containing (A, E) such that cl(U, E) ∩ (B, E) = ϕ.

(2) ⟹ (3)

Let (A, E) and (B, E) be two disjoint soft $I_{\pi g}$-closed subsets of X. By hypothesis There exists a soft open set (U, E) containing (A, E) such that cl(U, E) ∩ (B, E) = ϕ. Also cl(U, E) and (B, E) are disjoint $I_{\pi g}$-closed sets of X. Then there exists a soft open set (V, E) containing (B, E) such that cl(U, E) ∩ cl(V, E) = ϕ.

(3) ⟹ (1): Obvious

Theorem: 3.12

Let (X, τ, E, I) be a soft $I^*_{\pi g}$ - normal space. If (A, E) and (B, E) are disjoint soft $I_{\pi g}$-closed subsets of X, then there exist disjoint soft open sets (U, E) and (V, E) such that cl^*(A, E) ⊆ (U, E) and cl^*(B, E) ⊆ (V, E).

Proof:

Suppose that (A, E) and (B, E) are disjoint soft $I_{\pi g}$-closed sets. By theorem: 3.19 (3), there exists a soft open set (U, E) containing (A, E) and a soft open set (V, E) containing (B, E) such that

cl(U, E) ∩ cl(V, E) = ϕ. Since (A, E) is soft $I_{\pi g}$-closed set, (A, E) ⊆ (U, E) implies that cl^*(A, E) ⊆ (U, E). Similarly cl^*(B, E) ⊆ (V, E).

Theorem: 3.13

Let (X, τ, E, I) be a soft $I^*_{\pi g}$ - normal space. If (A, E) is soft $I_{\pi g}$-closed set and (B, E) is soft $I_{\pi g}$- open set containing (A, E), then there exists soft open set (U, E) such that (A, E) ⊆ cl^*(U, E) ⊆ (U, E) ⊆ int^*(B, E) ⊆ (B, E).

Proof:

Suppose (A, E) is soft $I_{\pi g}$-closed set and (B, E) is soft $I_{\pi g}$- open set containing (A, E). Since (A, E) and X − (B, E) are disjoint soft $I_{\pi g}$- closed sets, by theorem: 3.22 there exist disjoint soft open sets (U, E) and (V, E) such that cl^*(A, E) ⊆ (U, E) and cl^*(X − (B, E)) ⊆ (V, E). Now X – int^*(B, E) = cl^*(X − (B, E)) ⊆ (V, E) implies that X − (V, E) ⊆ int^*(B, E). Again (U ∩ V, E) = ϕ implies that (U, E) ⊆ X − (V, E). Hence (A, E) ⊆ cl^*(U, E) ⊆ (U, E) ⊆ X − (V, E) ⊆ int^*(B, E) ⊆ (B, E).

Definition: 3.14

A soft space (X, τ, E) is said to soft mildly normal space, if disjoint soft regular closed sets are separated by disjoint soft open sets.

Definition: 3.15

A subset of a soft ideal space (X, τ, E, I) is said to be soft I_{rg} - closed, if $(A, E)^*$ ⊆ (U, E) whenever (A, E) ⊆ (U, E) and (U, E) is soft regular open. The complement of soft I_{rg} - closed set is soft I_{rg} - open.

Lemma: 3.16

Let (X, τ, E, I) be a soft ideal space. A subset (A, E) ⊆ X is soft I_{rg} - open if and only if (F, E) ⊆ int^*(A, E) whenever (F, E) ⊆ (A, E) and (F, E) is soft regular closed.

Theorem: .17

Let (X, τ, E, I) be a soft ideal space, where I is soft completely codense. Then the following are equivalent:

(1) X is soft mildly normal

(2) For disjoint soft regular closed sets (A, E) and (B, E), there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

(3) For disjoint soft regular closed sets (A, E) and (B, E), there exist disjoint soft I_{rg} - open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

(4) For disjoint soft regular closed set (A, E) and soft regular open set (V, E) containing (A, E), there exists a soft I_{rg} -open set (U, E) of X such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆(V, E).

(5) For disjoint soft regular closed set (A, E) and soft regular open set (V, E) containing (A, E), there exists soft ∗-open set (U, E) of X such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆(V,E).

(6) For disjoint soft regular closed sets (A, E) and (B, E), there exist disjoint soft ∗-open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

Proof:

(1) ⟹ (2)

Suppose that (A, E) and (B, E) are disjoint soft regular closed sets. Since X is soft mildly normal, there exists disjoint soft open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E). But every soft open set is soft $I_{\pi g}$ -open set. Hence the proof.

(2) ⟹ (3)

The proof follows from the fact that every soft $I_{\pi g}$ -open set is soft I_{rg} -open set.

(3) ⟹ (4)

Suppose (A, E) is soft regular closed set and (B, E) is soft regular open set containing (A, E). Then (A, E) and X – (B, E) are disjoint soft regular closed sets. By hypothesis there exist disjoint soft I_{rg} -open set (U, E) and (V, E) such that (A, E) ⊆ (U, E) and X − (B, E) ⊆ (V, E). Since X − (B, E) is soft regular closed and (V, E) is soft I_{rg} -open, by lemma: 4.3 X − (B, E) ⊆ int^*(V, E). Thus cl^*(U, E) ⊆ X − int^*(V, E) ⊆ (B, E). Hence (U, E) is the required soft I_{rg} - open set such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆(B, E).

(4) ⟹ (5)

Let (A, E) be a soft regular closed set and (V, E) be a soft regular open set containing (A, E). Then there exists a soft I_{rg} - open set (G, E) of X such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ (V, E). By lemma: 4.3 (A, E) ⊆ int^*(G, E). If (U, E) = int^*(G, E) then (U, E) is a soft ⋆-open set and (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ cl^*(G, E) ⊆ (V, E). Hence (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆(V, E).

(5) ⟹ (6)

Let (A, E) and (B, E) be disjoint soft regular closed subsets of X. Then X − (B, E) is soft regular open set containing (A, E). By hypothesis there exists a soft ⋆-open set (U, E) of X such that (A, E) ⊆ (U, E) ⊆ cl^*(U, E) ⊆ X − (B, E). If (V, E) = X − cl^*(U, E) then (U, E) and (V, E) are disjoint soft ⋆-open set (U, E) of X such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E).

(6) ⟹ (1)

Let (A, E) and (B, E) be disjoint soft regular sets of X. Then there exist disjoint soft ⋆-open sets (U, E) and (V, E) such that (A, E) ⊆ (U, E) and (B, E) ⊆ (V, E). Since I is soft completely codense, $\tau^* \subseteq \tau^\alpha$ and so (U, E) and (V, E) are soft α-open sets. Hence (A, E) ⊆ (U, E) ⊆ int(cl(int(U, E))) = (G, E) and (B, E) ⊆ (V, E) ⊆ int(cl(int(V, E))) = (H, E). Hence (G, E) and (H, E) are the required disjoint soft open sets containing (A, E) and (B, E) respectively.

4. SOFT $I_{\pi g}$ - REGULAR SPACES

Definition: 4.1

A soft ideal space (X, τ, E, I) is said to be soft $I_{\pi g}$ - regular space, if for each pair consisting of a soft point x_e and a soft closed set (B, E) not containing x_e there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (V, E) such that x_e ∈ (U, E) and (B, E) ⊆ (V, E).

Preposition: 4.2

Every soft regular space is soft $I_{\pi g}$ - regular space, but the converse need not be true.

Proof:

Since every soft open set is soft $I_{\pi g}$ - open, every soft regular space is soft $I_{\pi g}$ - regular space.

Example: 4.3

X = {a, b, c, d} and E = {e_1, e_2}.

(A, E) = {(e_1, {c}), (e_2, {a})}

(B, E) = {(e_1, {d}), (e_2, {b})}

(C, E) = {(e_1, {c, d}), (e_2, {a, b})}

(D, E) = {(e_1, {a, d}), (e_2, {b, d})}

(F, E) = {(e_1, {b, c, d}), (e_2, {a, b,c})}

(G, E) = {(e_1, {a, c, d}), (e_2, {a, b, d})} where (A, E), (B, E), (C, E), (D, E), (F, E) and (G, E) soft sets over X and τ = {$\tilde{X}$, $\tilde{\phi}$, (A, E), (B, E), (C, E), (D, E) (F, E), (G, E)} is a soft topology over X.

Let I = {$\tilde{\phi}$, (I_1, E), (I_2, E), (I_3, E)} be a soft ideal over X, where (I_1, E) = {(e_1, {c}), (e_2,{a})}

(I_2, E) = {(e_1, {b}), (e_2, {c})}

(I_3, E) = {(e_1, {b, c}), (e_2, {a, c})}

Let x_e = c be a soft point in X and (H, E) = {(e_1, {a}), (e_2, {d})} be soft closed sets in X. Then there exist disjoint soft $I_{\pi g}$ - open sets (U, E) = {(e_1, {b, c, d}), (e_2, {a, b, c})} and (V, E) = {(e_1, {a}), (e_2, {d})} such that x_e ∈ (U, E) and (H, E) ⊆ (V, E). Hence X is soft $I_{\pi g}$ - normal space but not soft normal, because (V, E) is not soft open set in X.

Theorem: 4.4

In a soft ideal space (X, τ, E, I) the following are equivalent:

(1) X is soft $I_{\pi g}$ - regular space

(2) For every soft closed set (B, E) not containing $x_e \in$ X, there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (V, E) such that $x_e \in$ (U, E) and (B, E) ⊆ (V, E).

(3) For every soft open set (V, E) containing $x_e \in$ X, there exist disjoint soft $I_{\pi g}$ - open sets (U, E) of X such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

Proof:

(1) ⟹ (2): The proof follows from the definition of soft $I_{\pi g}$ - regular space.

(2) ⟹ (3)

Let (V, E) be a soft open subset such that $x_e \in$ (V, E). Then X − (V, E) is a soft closed set not containing x_e. Therefore there exist disjoint soft $I_{\pi g}$ - open sets (U, E) and (W, E) such that $x_e \in$ (U, E) and X − (V, E) ⊆ (W, E).Now X − (V, E) ⊆ (W, E) implies that X − (V, E) ⊆ int^*(W, E). Therefore X − int^*(W, E) ⊆ (V, E). Again (U ∩ W, E) = ϕ implies that (U, E) ∩ int^*(W, E) = ϕ. Hence cl^*(U, E) ⊆ X − int^*(W, E). Hence $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

(3) ⟹ (1)

Let (B, E) be a soft closed set not containing x_e. By hypothesis there exist a soft $I_{\pi g}$ -open set (U, E) such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ X − (B, E) . If (W, E) = X − cl^*(U, E) then (U, E) and (W, E) are disjoint soft $I_{\pi g}$ - open sets such that $x_e \in$ (U, E) and (B, E) ⊆ (W, E).

Theorem: 4.5

If (X, τ, E, I) is a soft $I_{\pi g}$ - regular, soft T_1- space where I is soft completely codense then X is soft regular.

Proof:

Let (B, E) be a soft closed set not containing $x_e \in$ X. By theorem: 5.4 there exist a soft $I_{\pi g}$ -open set (U, E) such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ X − (B, E) . Since X is soft T_1- space, {x_e} is soft closed and {x_e} ⊆ int^*(U, E). Since I is soft completely codense, $\tau^* \subseteq \tau^\alpha$ and so int^*(U, E) and X − cl^*(U, E) are soft α-open sets. Now $x_e \in int^*$(U, E) ⊆ int(cl(int(int^*(U, E)))) = (G, E) and (B, E) ⊆ X − cl^*(U, E) ⊆ int(cl(int(X − cl^*(U, E)))) = (H, E). Hence (G, E) and (H, E) are the disjoint soft open sets containing x_e and (B, E) respectively. Therefore X is soft regular.

Theorem: 4.6

If every soft open subset of a soft ideal space (X, τ, E, I) is soft ⋆-closed then (X, τ, E, I) is soft $I_{\pi g}$ - regular space.

Proof:

Suppose every soft open subset of X is soft ⋆-closed. Then every subset of X is soft $I_{\pi g}$ - closed. Hence every subset of X is soft $I_{\pi g}$ - open set. If (B, E) is a soft closed set not containing x_e then

{x_e} and (B, E) are the required disjoint soft $I_{\pi g}$ - open sets containing x_e and (B, E) respectively. Therefore X is soft $I_{\pi g}$ - regular.

Theorem: 4.7

Let (X, τ, E, I) be a soft ideal space where I is soft completely codense. Then the following are equivalent:

(1) X is soft regular

(2) For every soft closed set (A, E) and $x_e \in$ X – (A, E) there exist disjoint soft ⋆-open sets (U, E) and (V, E) such that $x_e \in$ (U, E) and (A, E) ⊆ (V, E).

(3) For every soft open set (V, E) of X and $x_e \in$ (V, E), there exists a soft ⋆-open set (U, E) such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

Proof:

(1) ⟹ (2)

Let (A, E) be a soft closed subset of X and let $x_e \in$ X – (A, E). Then there exist disjoint soft open sets (U, E) and (V, E) such that $x_e \in$ (U, E) and (A, E) ⊆ (V, E). But every soft open set is soft ⋆-open set. This gives the proof.

(2) ⟹ (3)

Let (V, E) be a soft open set containing $x_e \in$ X. Then X – (V, E) is soft closed and $x_e \in$ (V, E). By hypothesis there exist disjoint soft ⁎-open set (U, E) and (W, E) such that $x_e \in$ (U, E) and X − (V, E) ⊆ (W, E).Since (U ∩W, E) = ϕ, (U, E) ⊆ X – (W, E) and X – (W, E) is soft ⁎-closed. Thus cl^*(U, E) ⊆ X – (W, E) ⊆ (V, E). Therefore (U, E) is the required soft ⁎-open set such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

(3) ⟹ (1)

Let (A, E) be a soft closed set and $x_e \notin$ (A, E). By (3) there exists a soft ⁎-open set such that

$x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ X − (A, E). Let (V, E) = X − cl^*(U, E). Then (A, E) ⊆ (V, E) and (U, E) and (V, E) are disjoint soft ⁎-open sets. Since I is soft completely codense, (U, E) and (V, E) are soft α-open sets. Therefore (U, E) ⊆ int(cl(int(U, E))) = (G, E) and (A, E) ⊆ (V, E) ⊆ int(cl(int(V, E))) = (H, E). Then (G, E) and (H, E) are required disjoint soft open sets such that $x_e \in$ (G, E) and (A, E) ⊆ (H, E).Hence X is soft regular space.

Definition: 4.8

A soft space (X, τ, E) is said to be soft almost regular, if for each soft regular closed set (F, E) and a soft point $x_e \in$ X – (F, E), there exist disjoint soft open sets (U, E) and (V, E) such that $x_e \in$ (U, E) and (F, E) ⊆ (V, E).

Theorem: 4.9

Let (X, τ, E, I) be a soft ideal space where I is soft completely codense. Then the following are equivalent:

(1) X is soft almost regular

(2) For each soft regular closed set (A, E) and each $x_e \in$ X – (A, E) there exist disjoint soft ⁎-open sets (U, E) and (V, E) such that $x_e \in$ (U, E) and (A, E) ⊆ (V, E).

(3) For each soft regular open set (V, E) of X and $x_e \in$ (V, E), there exists a soft ⁎-open set (U, E) such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

Proof:

(1) ⟹ (2)

Let (A, E) be a soft regular closed and $x_e \in$ X – (A, E). Then there exist disjoint soft open sets (U, E) and (V, E) such that $x_e \in$ (U, E) and (A, E) ⊆ (V, E). But every soft open set is soft ⁎-open set. The proof follows.

(2) ⟹ (3)

Let (V, E) be a soft regular open set containing $x_e \in$ X. By (2) By hypothesis there exist disjoint soft ⁎-open set (U, E) and (W, E) such that $x_e \in$ (U, E) and X − (V, E) ⊆ (W, E). Since (U ∩W, E) = ϕ, cl^*(U, E) ⊆ X – (W, E) ⊆ (V, E). Therefore (U, E) is the required soft ⁎-open set such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ (V, E).

(3) ⟹ (1)

Let (A, E) be a soft regular closed set and $x_e \in$ X − (A, E). By hypothesis there exists a soft ⁎-open set such that $x_e \in$ (U, E) ⊆ cl^*(U, E) ⊆ X − (A, E). Let (V, E) = X − cl^*(U, E). Then (A, E) ⊆ (V, E) and (U, E) and (V, E) are disjoint soft ⁎-open sets. Since I is soft completely codense, (U, E) and (V, E) are soft α-open sets. Therefore $x_e \in$ (U, E) ⊆ int(cl(int(U, E))) = (G, E) and (A, E) ⊆ (V, E) ⊆ int(cl(int(V, E))) = (H, E). Then (G, E) and (H, E) are required disjoint soft open sets such that $x_e \in$ (G, E) and (A, E) ⊆ (H, E).Hence X is soft almost regular space.

5. CONCLUSIONS

In this paper we introduced the concept of soft $I_{\pi g}$ - regularity and soft $I_{\pi g}$ - normality in soft topological spaces and several properties concerning these spaces has been obtained. Furthermore we studied the behaviors of soft mildly normal spaces and soft almost regular spaces. We hope the findings of this paper will help the researchers for further studies on soft ideal topological spaces.

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