Fuzzy Neutrosophic Subgroupoids

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ABSTRACT— ***In this paper, we introduce the concept of fuzzy neutrosophic subgroupoids using fuzzy neutrosophic products. Further we investigate some properties of fuzzy neutrosophic products and subgroupoids.***

Keywords— Fuzzy neutrosophic point, fuzzy neutrosophic product, fuzzy neutrosophic subgroupoids, fuzzy neutrosophic ideals.

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1. INTRODUCTION

Neutrosophy is a branch of philosophy, which emphasizes the origin and nature of neutralites, along with their interaction with different conceptive domains. Fuzzy logic [1] extends classical logic by assigning a membership function ranging in degree between 0 and 1 to the variables. As a generalization of fuzzy logic, neutrosophic logic introduces a new component called indeterminacy and carries more information than fuzzy logic. The application of neutrosophic logic would lead to better performance than fuzzy logic. Neutrosophic logic is so new that its use in many fields merits exploration. The concept of Neutrosophic set was introduced by F. Smarandache [2,3,4,5,6]. It is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In neutrosophic logic a proposition has a degree of truth(T), a degree of indeterminacy(I) and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of] -0, 1+ [. Hence I. Arockiarani et.al [7] consider the neutrosophic set which takes the value from the subset of [0, 1]. Intuitionistic fuzzy sets [10,11] and interval valued intuitionistic fuzzy sets [12] can only handle incomplete information and inconsistent information which exist commonly in real situations. The focus of this paper is to initiate the concept of subgroupoids in fuzzy neutrosophic set. This paper elucidates the fuzzy neutrosophic subgroupoids and derives the results associated with it.

2. PRELIMINARIES

Definition 2.1: [7] A Fuzzy neutrosophic set A on the universe of discourse X is defined as

$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$ where $T, I, F : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

Definition 2.2: [7] Let X be a non- empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$

(i) $A \subseteq B$ for all x if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$.

(ii) $A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$.

(iii) $A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$.

(iv) A\B (x)= $\langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x)) \rangle$.

Definition 2.3: [7] A Fuzzy neutrosophic set A over the universe X is said to be null or empty Fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N .

Definition 2.4: [7] A Fuzzy neutrosophic set A over the universe X is said to be absolute (universe) Fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.5: [7] The complement of a Fuzzy neutrosophic set A is denoted by A^c and is defined as

$A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x)\rangle$ where $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x)$, $F_{A^c}(x) = T_A(x)$

The complement of a Fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

Definition 2.6: [8] Let X and Y be two non- empty sets and $f : X \to Y$ be a function.

(i) If $B = \{\langle y, T_B(y), I_B(y), F_B(y)\rangle : y \in Y\}$ is a fuzzy neutrosophic set in Y then the pre image of B under f ,denoted by $f^{-1}(B)$, is the fuzzy neutrosophic set in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x))\rangle : x \in X\}$$

Where $f^{-1}(T_B(x)) = T_B(f(x))$

(ii) If $A = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in X\}$ is a fuzzy neutrosophic set in X then the image of A under f ,denoted by $f(A)$,is the fuzzy neutrosophic set in Y defined by

$$f(A) = \{\langle y, f(T_A(y)), f(I_A(y)), f_{\sim}(F_A(y))\rangle : y \in Y\}$$ where

$$f(T_A(y)) = \begin{cases} \underset{x \in f^{-1}(y)}{Sup}\ T_A(x) & if\ f^{-1}(y) \neq 0_N \\ 0 & otherwise \end{cases}$$

$$f(I_A(y)) = \begin{cases} \underset{x \in f^{-1}(y)}{Sup}\ I_A(x) & if\ f^{-1}(y) \neq 0_N \\ 0 & otherwise \end{cases}$$

$$f_{\sim}(F_A(y)) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} F_A(x) & if\ f^{-1}(y) \neq 0_N \\ 1 & otherwise \end{cases}$$

And $f_{\sim}(F_A(y)) = (1 - f(1 - F_A))y$

Definition 2.7: [9] Let $(X, .)$ be a group and let A be fuzzy neutrosophic set in X . Then A is called a fuzzy neutrosophic group (in short, FNG) in X if it satisfies the following conditions: (i) $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$ (ii) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x), F_A(x^{-1}) \leq F_A(x)$

3. FUZZY NEUTROSOPHIC POINT AND FUZZY NEUTROSOPHIC PRODUCT

Definition 3.1: Let $p, q, r \in [0,1]$ and $p + q + r \leq 3$ .A fuzzy neutrosophic point $x_{(p,q,r)}$ of X is the fuzzy neutrosophic set in X defined by $x_{(p,q,r)}(y) = \begin{cases} (p,q,r) & if\ x = y \\ (0,0,1) & if\ y \neq x \end{cases}$,for each $y \in X$

Definition 3.2: A fuzzy neutrosophic point $x_{(p,q,r)}$ is said to belong to a fuzzy neutrosophic set $A = \langle T_A, I_A, F_A\rangle$ in X denoted by $x_{(p,q,r)} \in A$ if $p \leq T_A(x), q \leq I_A(x), r \leq F_A(x)$. We denote the set of all fuzzy neutrosophic points in X as $FNP(X)$.

Theorem 3.3: Let $A = \langle T_A, I_A, F_A\rangle$ and $B = \langle T_B, I_B, F_B\rangle$ be fuzzy neutrosophic sets in X ,then $A \subset B$ if and only if for each $x_{(p,q,r)} \in FNP(X)$, $x_{(p,q,r)} \in A \Rightarrow x_{(p,q,r)} \in B$.

Proof: Let $A \subseteq B$ and $x_{(p,q,r)} \in A$ then $p \leq T_A(x) \leq T_B(x), q \leq I_A(x) \leq I_B(x)$ and $r \geq F_A(x) \geq F_B(x)$ .Thus $x_{(p,q,r)} \in B$. Conversely, take $x_{(p,q,r)} \in FNP(X)$, $x_{(p,q,r)} \in A \Rightarrow x_{(p,q,r)} \in B$ and $x \in X$.
Let $p = T_A(x), q = I_A(x), r = F_A(x)$ .Then $x_{(p,q,r)}$ is a fuzzy neutrosophic point in X and $x_{(p,q,r)} \in A$.
By the hypothesis, $x_{(p,q,r)} \in B$ .Thus $T_A(x) = p \leq T_B(x), I_A(x) = q \leq I_B(x), F_A(x) = r \geq F_B(x)$ .Hence $A \subseteq B$.

Theorem3.4: Let $A=\langle T_A, I_A, F_A\rangle$ be a fuzzy neutrosophic set of X .Then $A=\bigcup\{x_{(p,q,r)}, x_{(p,q,r)}\in A\}$.

Definition3.5: Let X be a set and let $p,q,r\in[0,1]$ with $p+q+r\le 3$ .Then the fuzzy neutrosophic support $C_{(p,q,r)}\in X$ is defined by for each

$x\in X$, $C_{(p,q,r)}(x)=(p,q,r)$.(i.e.,) $T_{C_{(p,q,r)}}(x)=p, I_{C_{(p,q,r)}}(x)=q, F_{C_{(p,q,r)}}(x)=r$.

Definition 3.6: [3]Let $(X,.)$ be a groupoid and let A and B be two fuzzy neutrosophic sets in X .Then the fuzzy neutrosophic product of A and B , $A\circ B$,is defined as follows: for any $x\in X$,

$$T_{A\circ B}(x)=\begin{cases}\bigvee_{yz=x}[T_A(y)\wedge T_B(z)] & for\ each\ (y,z)\in X\times X\ with\ yz=x,\\ 0 & otherwise\end{cases}$$

$$I_{A\circ B}(x)=\begin{cases}\bigvee_{yz=x}[I_A(y)\wedge I_B(z)] & for\ each\ (y,z)\in X\times X\ with\ yz=x,\\ 0 & otherwise\end{cases}$$

$$F_{A\circ B}(x)=\begin{cases}\bigwedge_{yz=x}[F_A(y)\vee F_B(z)] & for\ each\ (y,z)\in X\times X\ with\ yz=x,\\ 1 & otherwise\end{cases}$$

Proposition 3.7:

Let "$\circ$" be as above, let $x_{(\alpha,\beta,\gamma)}, y_{(\alpha',\beta',\gamma')}$ be two fuzzy neutrosophic points and let $A,B\in FNS(X)$ .Then (1) $x_{(\alpha,\beta,\gamma)}\circ y_{(\alpha',\beta',\gamma')}=(xy)_{(\alpha\wedge\alpha',\beta\wedge\beta',\gamma\vee\gamma')}$ (2) $A\circ B=\bigcup_{x_{(\alpha,\beta,\gamma)}\in A, y_{(\alpha',\beta',\gamma')}\in B} x_{(\alpha,\beta,\gamma)}\circ y_{(\alpha',\beta',\gamma')}$.

Proof: Let $z\in X$ .Then

$$T_{x_{(\alpha,\beta,\gamma)},y_{(\alpha',\beta',\gamma')}}(z)=\begin{cases}\bigvee_{x'y'=z}\left[T_{x_{(\alpha,\beta,\gamma)}}(x')\wedge T_{y_{(\alpha',\beta',\gamma')}}(y')\right] & for\ each\ (x',y')\in X\times X\ with\ x'y'=z\\ 0 & otherwise\end{cases}$$

$$=\begin{cases}\alpha\wedge\alpha' & if\ \ xy=z\\ 0 & otherwise\end{cases}$$

Similarly

$$I_{x_{(\alpha,\beta,\gamma)},y_{(\alpha',\beta',\gamma')}}(z)=\begin{cases}\beta\wedge\beta' & if\ \ xy=z\\ 0 & otherwise\end{cases}.$$

$$F_{x_{(\alpha,\beta,\gamma)},y_{(\alpha',\beta',\gamma')}}(z)=\begin{cases}\bigwedge_{x'y'=z}\left[F_{x_{(\alpha,\beta,\gamma)}}(x')\vee F_{y_{(\alpha',\beta',\gamma')}}(y')\right] & for\ each\ (x',y')\in X\times X\ with\ x'y'=z\\ 1 & otherwise\end{cases}$$

$$=\begin{cases}\gamma\vee\gamma' & if\ \ xy=z\\ 1 & otherwise\end{cases}$$

Hence $x_{(\alpha,\beta,\gamma)}\circ y_{(\alpha',\beta',\gamma')}=(xy)_{(\alpha\wedge\alpha',\beta\wedge\beta',\gamma\vee\gamma')}$.

(2) Let $C=\bigcup_{x_{(\alpha,\beta,\gamma)}\in A, y_{(\alpha',\beta',\gamma')}\in B} x_{(\alpha,\beta,\gamma)}\circ y_{(\alpha',\beta',\gamma')}$ .Let $w\in X$ and we may assume that there exist $u,v\in X$ such that $uv=w$, $T_A(u)\neq 0, I_A(u)\neq 0, F_A(u)\neq 1$ and $T_B(u)\neq 0, I_B(u)\neq 0, F_B(u)\neq 1$.

$$T_{A\circ B}(w) = \bigvee_{uv=w} [T_A(u) \wedge T_B(v)]$$
$$\geq \bigvee_{uv=w} \bigvee_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} \left[T_{x_{(\alpha,\beta,\gamma)}}(u) \wedge T_{y_{(\alpha',\beta',\gamma')}}(v)\right] = T_C(w)$$

Since $u_{(T_A(u),I_A(u),F_A(u))} \in A$ and $v_{(T_B(u),I_B(u),F_B(u))} \in B$

$$T_C(w) = \bigvee_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} \bigvee_{uv=w} \left[T_{x_{(\alpha,\beta,\gamma)}}(u) \wedge T_{y_{(\alpha',\beta',\gamma')}}(v)\right]$$
$$= \bigvee_{uv=w} \left(\bigvee_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} \left[T_{x_{(\alpha,\beta,\gamma)}}(u) \wedge T_{y_{(\alpha',\beta',\gamma')}}(v)\right] \right)$$
$$\geq \bigvee_{uv=w} \left[T_{u(T_A(u),I_A(u),F_A(u))}(u) \wedge T_{v(T_A(v),I_A(v),F_A(v))}(v)\right] = \bigvee_{uv=w} [T_A(u) \wedge T_B(u)] = T_{A\circ B}(w)$$

Thus $T_{A\circ B} = T_C$

Similarly, $I_{A\circ B} = I_C$

$$F_{A\circ B}(w) = \bigwedge_{uv=w} [F_A(u) \vee F_B(v)]$$
$$\leq \bigwedge_{uv=w} \bigwedge_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} \left[F_{x_{(\alpha,\beta,\gamma)}}(u) \vee F_{y_{(\alpha',\beta',\gamma')}}(v)\right] = F_C(w)$$

Since $u_{(T_A(u),I_A(u),F_A(u))} \in A$ and $v_{(T_B(u),I_B(u),F_B(u))} \in B$

$$F_C(w) = \bigwedge_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} \bigwedge_{uv=w} \left[F_{x_{(\alpha,\beta,\gamma)}}(u) \vee F_{y_{(\alpha',\beta',\gamma')}}(v)\right]$$
$$= \bigwedge_{uv=w} \left(\bigwedge_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} \left[F_{x_{(\alpha,\beta,\gamma)}}(u) \vee F_{y_{(\alpha',\beta',\gamma')}}(v)\right] \right)$$
$$\leq \bigwedge_{uv=w} \left[F_{u(T_A(u),I_A(u),F_A(u))}(u) \vee F_{v(T_A(v),I_A(v),F_A(v))}(v)\right] = \bigwedge_{uv=w} [F_A(u) \vee F_B(u)] = F_{A\circ B}(w)$$

Thus $F_{A\circ B} = F_C$ .Hence $A \circ B = \bigcup_{x_{(\alpha,\beta,\gamma)} \in A, y_{(\alpha',\beta',\gamma')} \in B} x_{(\alpha,\beta,\gamma)} \circ y_{(\alpha',\beta',\gamma')}$.

The following proposition holds from definition 3.6.

Proposition 3.7: Let $(X,.)$ be a groupoid and let "$\circ$" be as above.

(1) If "." is associative (respectively commutative) in X ,then so is "$\circ$" in $FNS(X)$.

(2) If "." has a unity $e \in X$,then $e_{(1,1,0)} \in FNP(X)$ is a unity of "$\circ$" in $FNS(X)$.

(i.e.,) $A \circ e_{(1,1,0)} = A = e_{(1,1,0)} \circ A$ for each $A \in FNS(X)$.

Proof: Proof is immediate.

4. FUZZY NEUTROSOPHIC SUBGROUPOIDS AND IDEALS

Definition 4.1: Let $(G,.)$ be a groupoid and let $0_N \neq A \in FNS(G)$.Then A is called a fuzzy neutrosophic subgroupoid in G (in short, $FNSGP$ in G) if $A \circ A \subset A$.

Definition 4.2: Let $(G,.)$ be a groupoid and let $A \in FNS(X)$ .Then A is called a fuzzy neutrosophic subgroupoid in G (in short , $FNSGP$ in G) if for any $x, y \in G$, $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$ and

$F_A(xy) \le F_A(x) \vee F_A(y)$ .It is clear that 0_N and 1_N are both $FNSGP$s of G .

The following are the immediate results of Definition 3.6 and Definition 4.1.

Proposition 4.3: Let $(G,.)$ be a groupoid and let $0_N \ne A \in FNS(G)$ .Then the following conditions are equivalent:

(1) A is a $FNSGP$ in G .

(2) For any $x_{(\alpha,\beta,\gamma)}, y_{(\alpha',\beta',\gamma')} \in A, x_{(\alpha,\beta,\gamma)} \circ y_{(\alpha',\beta',\gamma')} \in A,$

(i.e.,) $(A,\circ)$ is a groupoid.

(3) For any $x, y \in X$ $T_A(xy) \ge T_A(x) \wedge T_A(y), I_A(xy) \ge I_A(x) \wedge I_A(y)$ and $F_A(xy) \le F_A(x) \vee F_A(y)$

Proposition 4.4: Let A be a $FNSGP$ in a groupoid $(G,.)$.

(1) If "." is associative in G ,then so is " $\circ$ " in A ,(i.e.,) for any $x_{(\alpha,\beta,\gamma)}, y_{(\alpha',\beta',\gamma'),} z_{(\alpha'',\beta'',\gamma'')} \in A,\ (x_{(\alpha,\beta,\gamma)} \circ y_{(\alpha',\beta',\gamma')}) \circ z_{(\alpha'',\beta'',\gamma'')} = x_{(\alpha,\beta,\gamma)} \circ (y_{(\alpha',\beta',\gamma')} \circ z_{(\alpha'',\beta'',\gamma'')})$

(2) If "." is commutative in G ,then so is " $\circ$ " in A ,(i.e.,) for any $x_{(\alpha,\beta,\gamma)}, y_{(\alpha',\beta',\gamma')} \in A,\ x_{(\alpha,\beta,\gamma)} \circ y_{(\alpha',\beta',\gamma')} = y_{(\alpha',\beta',\gamma')} \circ x_{(\alpha,\beta,\gamma)}$.

(3) If "." has a unity $e \in G$,then $e_{(1,1,0)} \circ x_{(\alpha,\beta,\gamma)} = x_{(\alpha,\beta,\gamma)} = x_{(\alpha,\beta,\gamma)} \circ e_{(1,1,0)}$ for each $x_{(\alpha,\beta,\gamma)} \in A$.

Proof: Proof is obvious.

Definition 4.5: Let G be a groupoid and let $A \in FNS(G)$ .Then A is called a:

(1) fuzzy neutrosophic left ideal (in short $FNLI$) of G if for any $x, y \in G$, $A(xy) \ge A(y)$.(i.e.,) $T_A(xy) \ge T_A(y), I_A(xy) \ge I_A(y)$ and $F_A(xy) \le F_A(y)$

(2) fuzzy neutrosophic right ideal (in short $FNRI$) of G if for any $x, y \in G$, $A(xy) \ge A(x)$.(i.e.,) $T_A(xy) \ge T_A(x), I_A(xy) \ge I_A(x)$ and $F_A(xy) \le F_A(x)$

(3) fuzzy neutrosophic ideal (in short FNI) of G if it is both a $FNLI$ and $FNRI$

It is clear that A is a FNI of G if and only if for any $x, y \in G, T_A(xy) \ge T_A(x) \vee T_A(y), I_A(xy) \ge I_A(x) \vee I_A(y)$ and $F_A(xy) \le F_A(x) \wedge F_A(y)$ .Moreover ,a FNI (respectively $FNLI$, $FNRI$) is a $FNSGP$ of G .Note that for any $FNSGP$ A of G we have $T_A(x^n) \ge T_A(x), I_A(x^n) \ge I_A(x)$ and $F_A(x^n) \le F_A(x)$ for each $x \in G$, where x^n is any composite of x 's.

We will denote the set of all $FNSGP$s of G as $FNSGP(G)$.

Definition 4.6: Let A be a fuzzy neutrosophic set in X and let $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \le 3$ .Then the set $X_A^{(\lambda,\mu,\nu)} = \{x \in X : A(x) \ge C_{(\lambda,\mu,\nu)}(x)\} = \{x \in X : T_A(x) \ge \lambda, I_A \ge \mu, F_A \le \nu\}$ is called a $(\lambda, \mu, \nu) -$ level subset of A .

Proposition 4.7: Let G be a groupoid and let $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \le 3$ .If A is a FNI (respectively $FNLI$, $FNRI$) of G ,then $G_A^{(\lambda,\mu,\nu)}$ is a subgroupoid or a (left, right) ideal of G .

Proof: Suppose $A \in FNSGP(G)$ and let $x, y \in G_A^{(\lambda,\mu,\nu)}$ .Then $T_A(x) \ge \lambda, I_A(x) \ge \mu, F_A(x) \le \nu$ and $T_A(y) \ge \lambda, I_A(y) \ge \mu, F_A(y) \le \nu$ .Since $A \in FNSGP(G)$, $T_A(xy) \ge T_A(x) \wedge T_A(y), I_A(xy) \ge I_A(x) \wedge I_A(y), F_A(xy) \le F_A(x) \vee F_A(y)$.

Thus $T_A(xy) \geq \lambda, I_A(xy) \geq \mu, F_A(xy) \leq \nu$ .So $xy \in G_A^{(\lambda,\mu,\nu)}$ .Hence $G_A^{(\lambda,\mu,\nu)}$ is a left ideal of G .By the similar argument,we can easily check that $G_A^{(\lambda,\mu,\nu)}$ is a (right) ideal of G .This completes the proof.

Proposition 4.8: Let $\{A_\alpha\}_{\alpha\in\Gamma} \subset FNSGP(G)$ .Then $\bigcap_{\alpha\in\Gamma} A_\alpha \in FNSGP(G)$.

Proof: Let $A = \bigcap_{\alpha\in\Gamma} A_\alpha$ and let $x, y \in G$ .Then $A=\left(\wedge_{\alpha\in\Gamma} T_{A_\alpha}, \wedge_{\alpha\in\Gamma} I_{A_\alpha}, \vee_{\alpha\in\Gamma} F_{A_\alpha}\right)$.

$$T_A(xy) = \wedge_{\alpha\in\Gamma} T_{A_\alpha}(xy) \geq \wedge_{\alpha\in\Gamma}\left[T_{A_\alpha}(x) \wedge T_{A_\alpha}(y)\right] = \left(\wedge_{\alpha\in\Gamma} T_{A_\alpha}(x)\right) \wedge \left(\wedge_{\alpha\in\Gamma} T_{A_\alpha}(y)\right) = T_A(x) \wedge T_A(y).$$

Similarly, $I_A(xy) \geq I_A(x) \wedge I_A(y)$

$$F_A(xy) = \vee_{\alpha\in\Gamma} F_{A_\alpha}(xy) \leq \vee_{\alpha\in\Gamma}\left[F_{A_\alpha}(x) \vee F_{A_\alpha}(y)\right] = \left(\vee_{\alpha\in\Gamma} F_{A_\alpha}(x)\right) \vee \left(\vee_{\alpha\in\Gamma} F_{A_\alpha}(y)\right) = F_A(x) \vee F_A(y)$$

Hence $\bigcap_{\alpha\in\Gamma} A_\alpha$ is a $FNSGP$ of G .

Proposition 4.9: Let $\{A_\alpha\}_{\alpha\in\Gamma}$ be any family of FNI s $(FNLIs, FNRIs)$ .Then $\bigcap_{\alpha\in\Gamma} A_\alpha$ or $\bigcup_{\alpha\in\Gamma} A_\alpha$ is a $FNI(FNLI, FNRI)$.

Proof: Let G be a groupoid and let $\{A_\alpha\}_{\alpha\in\Gamma}$ be any family of $FNIs(FNLIs, FNRIs)$ of G .Let $A = \bigcap_{\alpha\in\Gamma} A_\alpha$ and let $x, y \in G$.

Suppose $\{A_\alpha\}_{\alpha\in\Gamma}$ is a family of $FNLI$ s of G.Then $T_A(xy) = \vee_{\alpha\in\Gamma} T_{A_\alpha}(xy) \geq \wedge_{\alpha\in\Gamma} T_{A_\alpha}(y)$.(Since A_α is a $FNLI$ of G for each $\alpha \in \Gamma$). Similarly, $I_A(xy) = \vee_{\alpha\in\Gamma} I_{A_\alpha}(xy) \geq \wedge_{\alpha\in\Gamma} I_{A_\alpha}(y)$.

$F_A(xy) = \vee_{\alpha\in\Gamma} F_{A_\alpha}(xy) \leq \vee_{\alpha\in\Gamma} F_{A_\alpha}(y)$. (Since A_α is a $FNLI$ of G for each $\alpha \in \Gamma$)

So $A = \bigcap_{\alpha\in\Gamma} A_\alpha$ is a $FNLI$ of G .By the similar arguments, we can easily check that the remainders hold. Also we can see that $\bigcup_{\alpha\in\Gamma} A_\alpha$ is a $FNI(FNLI, FNRI)$ .This completes the proof.

5. HOMOMORPHISMS

Proposition 5.1: Let $f : G \to G''$ be a groupoid homomorphism and let $B \in FNS(G'')$

(1) If $B \in FNSGP(G'')$,then $f^{-1}(B) \in FNSGP(G)$.

(2) If B is a $FNI(FNLI, FNRI)$ of G'' then $f^{-1}(B)$ is a $FNI(FNLI, FNRI)$ of G .

Proof: (1) By definition 2.6, $f^{-1}(B) = \left(f^{-1}(T_B), f^{-1}(I_B), f^{-1}(F_B)\right)$ where $f^{-1}(T_B(x)) = T_B(f(x))$.

Let $x, y \in G$ .Then $T_{f^{-1}(B)}(xy) = f^{-1}(T_B(xy)) = T_B(f(xy))$

$= T_B(f(x)f(y))$ (since f is a groupoid homomorphism)

$\geq T_B(f(x)) \wedge T_B(f(y)) = f^{-1}(T_B(x)) \wedge f^{-1}(T_B(y))$

Similarly, $I_{f^{-1}(B)}(xy) \geq f^{-1}(I_B(x)) \wedge f^{-1}(I_B(y))$.

$$F_{f^{-1}(B)}(xy) = f^{-1}(F_B(xy)) = F_B(f(xy))$$

$$= F_B(f(x)f(y)) \text{ (since } f \text{ is a groupoid homomorphism)}$$

$$\leq F_B(f(x)) \wedge F_B(f(y)) = f^{-1}(F_B(x)) \wedge f^{-1}(F_B(y)).$$

Hence $f^{-1}(B) \in FNSGP(G)$.

(2) By the similar arguments of the proof of **(1)**, it is clear.

Definition 5.2: Let $A \in FNS(G)$.Then A is said to have the sup property if for any $T \in P(G)$,there exists a $t_0 \in T$ such that $A(t_0) = \bigcup_{t \in T} A(t)$ .i.e., $T_A(t_0) = \vee_{t \in T} T_A(t), I_A(t_0) = \vee_{t \in T} I_A(t), F_A(t_0) = \wedge_{t \in T} F_A(t)$,where $P(G)$ denotes the power set of G.

Remark 5.3: Let $A \in FNS(G)$.If A can take only finitely many values (in particular, if they are characteristic function), then A has the sup property.

Proposition 5.4: Let $f : G \rightarrow G''$ be a groupoid homomorphism and let $A \in FNS(G)$ have the sup property.

(1) If $A \in FNSGP(G)$,then $f(A) \in FNSGP(G'')$.

(2) If A is a $FNI(FNLI, FNRI)$ of G,then $f(A)$ is a $FNI(FNLI, FNRI)$ of G''.

Proof: (1) Let $y, y' \in G''$.Then we can consider four cases:

(i) $f^{-1}(y) \neq \phi, f^{-1}(y') \neq \phi$

(ii) $f^{-1}(y) \neq \phi, f^{-1}(y') = \phi$

(iii) $f^{-1}(y) = \phi, f^{-1}(y') \neq \phi$

(iv) $f^{-1}(y) = \phi, f^{-1}(y') = \phi$

We prove only the case (i) and omit the remainders. Since A has the sup property, there exist $x_0 \in f^{-1}(y)$ and $x_0' \in f^{-1}(y')$ such that:

$$T_A(x_0) = \vee_{x \in f^{-1}(y)} \left(T_A(x_0), I_A(x_0), F_A(x_0)\right) = \vee_{x \in f^{-1}(y)} T_A(x) \text{ and}$$

$$T_A(x_0') = \vee_{x \in f^{-1}(y)} \left(T_A(x_0'), I_A(x_0'), F_A(x_0')\right) = \vee_{x \in f^{-1}(y)} T_A(x').$$

Then: $T_{f(A)}(yy') = f(T_A)(yy')$

$$= \vee_{x \in f^{-1}(yy')} T_A(z) \geq T_A(x_0 x_0') \geq T_A(x_0) \wedge T_A(x_0') = \left(\vee_{x \in f^{-1}(y)} T_A(x)\right) \wedge \left(\vee_{x \in f^{-1}(y)} T_A(x')\right)$$

$$= f(T_A(y)) \wedge f(T_A(y'))$$

Similarly, $I_{f(A)}(yy') = f(I_A(y)) \wedge f(I_A(y'))$.

$$F_{f(A)}(yy') = f(F_A)(yy')$$

$$= \wedge_{x \in f^{-1}(yy')} F_A(z) \leq F_A(x_0 x_0') \leq F_A(x_0) \vee F_A(x_0') = \left(\wedge_{x \in f^{-1}(y)} F_A(x)\right) \vee \left(\wedge_{x \in f^{-1}(y)} F_A(x')\right)$$

$= f(F_A(y)) \vee f(F_A(y'))$.

(2) Proof is similar to the proof of (1).

Definition 5.5: Let $f: X \to Y$ be a mapping and let A be a fuzzy neutrosophic set in X .Then A is said to be $f-$invariant if $f(x) = f(y) \Rightarrow A(x) = A(y)$,(i.e.,) $T_A(x) = T_A(y), I_A(x) = I_A(y), F_A(x) = F_A(y)$.

It is clear that if A is $f-$invariant, then $f^{-1}(f(A)) = A$.

Proposition 5.6: Let $f: X \to Y$ be a mapping and let $A = \{A \in FNS(X) : A\ is\ f-in\mathrm{var}ient\}$ .Then f is a one - to-one correspondence between A and $FNS(f(X))$.

Proof: Proof follows from the definition 5.5.

Corollary 5.7: Let $f: G \to G''$ be a mapping and let $A = \{A \in FNSGP(G) : A\,is$ invariant and has sup property}. Then f is a one-to-one correspondence between A and $FNSGP(f(G))$.

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