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A new method for finding Root of Nonlinear Equations by using Nonlinear Regression

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ABSTRACT— ***In this paper, we describe a new method for solving nonlinear equations by using nonlinear regression. The method is verified on a number of test examples and numerical results obtained show that the proposed method is faster than Bisection method and Regula Falsi method and equal modified Regula Falsi method.***

Keywords— Nonlinear equations, Nonlinear regression, Roots.

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1. INTRODUCTION

We now consider that most basic problems in scientific and engineering work is to locate a real root, α, of a nonlinear equation

$$f(x) = 0, \tag{1}$$

where f denote a continuously differentiable on $[a,b] \subset \Re$ and has a least one root, α, in $[a,b]$. Generally algorithms for solving (1) can be divided into main groups which are direct methods and iterative methods. Direct methods can be completed in exact root but often it will not be possible to solve some the equations. Iterative methods are methods which converge to the root over time. These algorithms run until some convergence criterion is met. When choosing this method to use one important consideration is how quickly the algorithm converges to the root or solution that we call method's convergence rate.

Some of the more classical numerical methods for solving nonlinear equations without using derivatives include bisection, secant and regula falsi (see [4]). Many of these methods, such as the regula falsi method, are fast in general but can be extremely slow for certain classes of functions and are likely to fail if the initial starting value is not sufficiently close to α. On the other hand, bisection is a safe method always convergence but its rate of convergence is slow. A successful algorithm developed by Neamvonk [3] has been modified the regula falsi method to improved its order of convergence.

The regula falsi method, it can be well known that the idea is to defined $[a,b]$ and find a linear function on $(a, f(a))$ and $(b, f(b))$ to estimate the root of (1). Due to the nonlinear equation, f , if we can find a nonlinear function on 3 initial value which are $(a, f(a))$, $(b, f(b))$ and $(c, f(c))$ where $c = \frac{a+b}{2}$ then this procedure may lead to the root quicker and more accurate logarithm function is widely used in

$$\hat{y}_i = C + A\ln x_i \,. \tag{2}$$

The function (2) can be applied where $A > 0$ and $A < 0$.

2. NEW METHOD

In fitting nonlinear function we define initial point a,b,c and their function $f(a), f(b)$ and $f(c)$. The general form of logarithm function [1] is

$$y_i = \alpha \ln(\beta x_i) + e_i \,, \tag{3}$$

where α and β are the unknown constants. The function (3) can be rewritten as

$$y_i = \alpha(\ln\beta + \ln x_i) + e_i\,, \tag{4}$$

and

$$y_i = \theta + \alpha \ln x_i + e_i\,, \tag{5}$$

where $\theta = \alpha \ln\beta$ is a constant. The constants in (5) can be estimated using least square method [2] i.e. the sum of square error is minimum. Therefore, the estimated logarithm function is

$$\hat{y}_i = C + A\ln x_i\,, \tag{6}$$

where A and C are the least square estimated values of α and θ respectively.

Let e is different between the true value, y_i, and the estimated value, $\hat{y}_i$. Therefore,

$$\hat{e}_i = y_i - \hat{y}_i\,. \tag{7}$$

The sum of square error,

$$\sum_{i=1}^{n}\hat{e}_i^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{8}$$

and substitute (6) in (8) will get

$$\sum\hat{e}_i^2 = \sum(y_i - C - A\ln x_i)^2\,. \tag{9}$$

We will find A and C that minimize this function. The derivative with respect to A and C are applied and set to 0. So,

$$\frac{\partial\sum\hat{e}_i^2}{\partial C} = -2\sum(y_i - C - A\ln x_i) = 0, \tag{10}$$

$$\frac{\partial\sum\hat{e}_i^2}{\partial A} = -2\sum(y_i - C - A\ln x_i)\ln x_i = 0. \tag{11}$$

From (10) and (11), we have

$$\sum y_i = nC + A\sum\ln x_i\,, \tag{12}$$

$$\sum y_i \ln x_i = nC\sum\ln x_i, + A\sum(\ln x_i)^2. \tag{13}$$

Then, multiply $\sum_{i=1}^{n}\ln x_i$ in both side of (12) and n in both side of (13). We have

$$\sum y\sum\ln x_i = nC\sum\ln x_i + A\left(\sum\ln x_i\right)^2, \tag{14}$$

$$n\sum y\sum\ln x_i = nC\sum\ln x_i + nA\left(\sum\ln x_i\right)^2. \tag{15}$$

Subtract (15) by (14), we have

$$A = \frac{\sum y_i\ln x_i - \frac{1}{n}\sum y_i\sum\ln x_i}{\sum(\ln x_i)^2 - \frac{1}{n}\left(\sum\ln x_i\right)^2}, \tag{16}$$

and

$$C = \tfrac{1}{n}\sum y_i - \tfrac{A}{n}\sum\ln x_i\,. \tag{17}$$

The intercept x-axis of the equation (6) at point x which can be estimated by

$$x = \exp\{-\frac{C}{A}\} \tag{18}$$

This method is applied to find root of nonlinear equation by defining the y as $f(x)$ and range of a, b and c where $c = \frac{a+b}{2}$. In this research, the points of $(a, f(a))$, $(b, f(b))$, and $(c, f(c))$ are applied in (16) and (17) as shown in Figure 1.

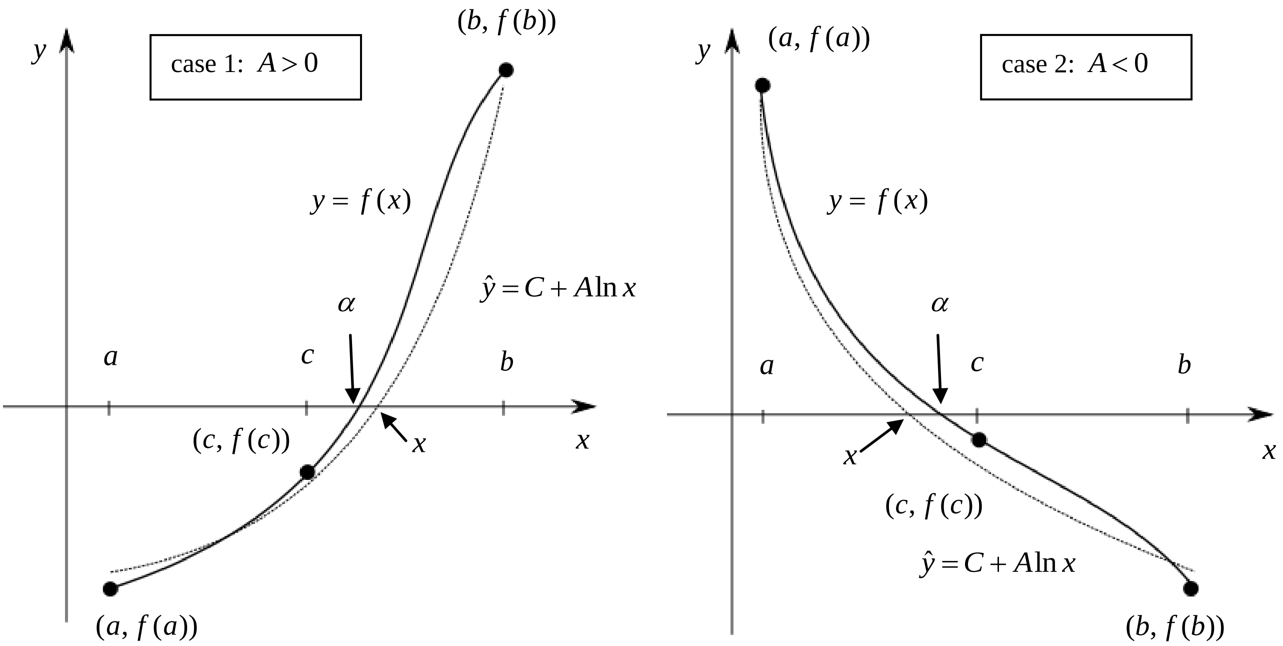

Figure 1: graph of $y = f(x)$ with initial point a,b and c. Dotted line represents approximate function $\hat{y}=C+A\ln x$ in case 1 $A>0$ and case 2 $A<0$.

3. ALGORITHM

The new method have four steps:

(i) Set $[a,b]$ is an initial interval which has at least a root in the interval,

(ii) compute $c=\dfrac{a+b}{2}$,

$$A=\frac{\left(f(a)\ln(a)+f(b)\ln(b)+f(c)\ln(c)\right)-\left(\left(f(a)+f(b)+f(c)\right)\left(\ln(a)+\ln(b)+\ln(c)\right)/3\right.}{\left((\ln(a))^2+(\ln(b))^2+(\ln(c))^2\right)-\left(\ln(a)+\ln(b)+\ln(c)\right)^2/3},$$

$$C=\frac{f(a)+f(b)+f(c)}{3}-A\frac{\ln(a)+\ln(b)+\ln(c)}{3} ,$$

and $x=e^{-\frac{C}{A}}$,

(iii) replace the interval $[a,b]$ with $[x,b]$ if $f(a)f(x)>0$ or with $[a,x]$ if $f(a)f(x)<0$,

(iv) return step (ii) until absolute error $|f(x)|<\varepsilon$.

4. RESULTS

Table 1 presents comparison of iteration numbers and running time between Bisection (BS), Regula Falsi (RF), modified Regula Falsi (MRF), and present work (NEW) with $\varepsilon = 1\times10^{-10}$ and initial interval $[0.1,1]$ in four equations. The results show that the BS method presents the biggest number of iterations and time in all cases. The RF methods provides more than 10 iterations. However, the MRF and New methods have fairly similar results. The MRF have the smallest number of iterations whereas the NEW method have the smallest time in most cases.

Table 1: The number of iteration and running time of BS, RF, MRF and NEW methods for specifics precision

Equation	Method	No. Iteration	Time (s)	Approximate root
$f_1(x) = x^2 - (1-x)^5 = 0$	BS	33	1.062	0.345954815842023
	RF	20	0.843	
	MRF	5	0.578	
	NEW	6	0.375	
$f_2(x) = \cos(x) - x^3 = 0$	BS	34	1.118	0.8654740331016205
	RF	12	0.531	
	MRF	5	0.594	
	NEW	6	0.391	
$f_3(x) = xe^x - 1 = 0$	BS	33	1.094	0.5671432904097837
	RF	20	0.906	
	MRF	5	0.516	
	NEW	6	0.547	
$f_4(x) = e^{-e^{-x}} - x = 0$	BS	29	0.953	0.5671432904012503
	RF	8	0.344	
	MRF	6	0.609	
	NEW	7	0.453	

Figure 2 shows the absolute error, absolute error, $|f(x)|$, of the four methods for the first 6 iterations. It can be seen that the error of the Bisection method are frustrated along the iteration. While the errors of the RF, MRF and NEW method decrease when the number of iteration increases.

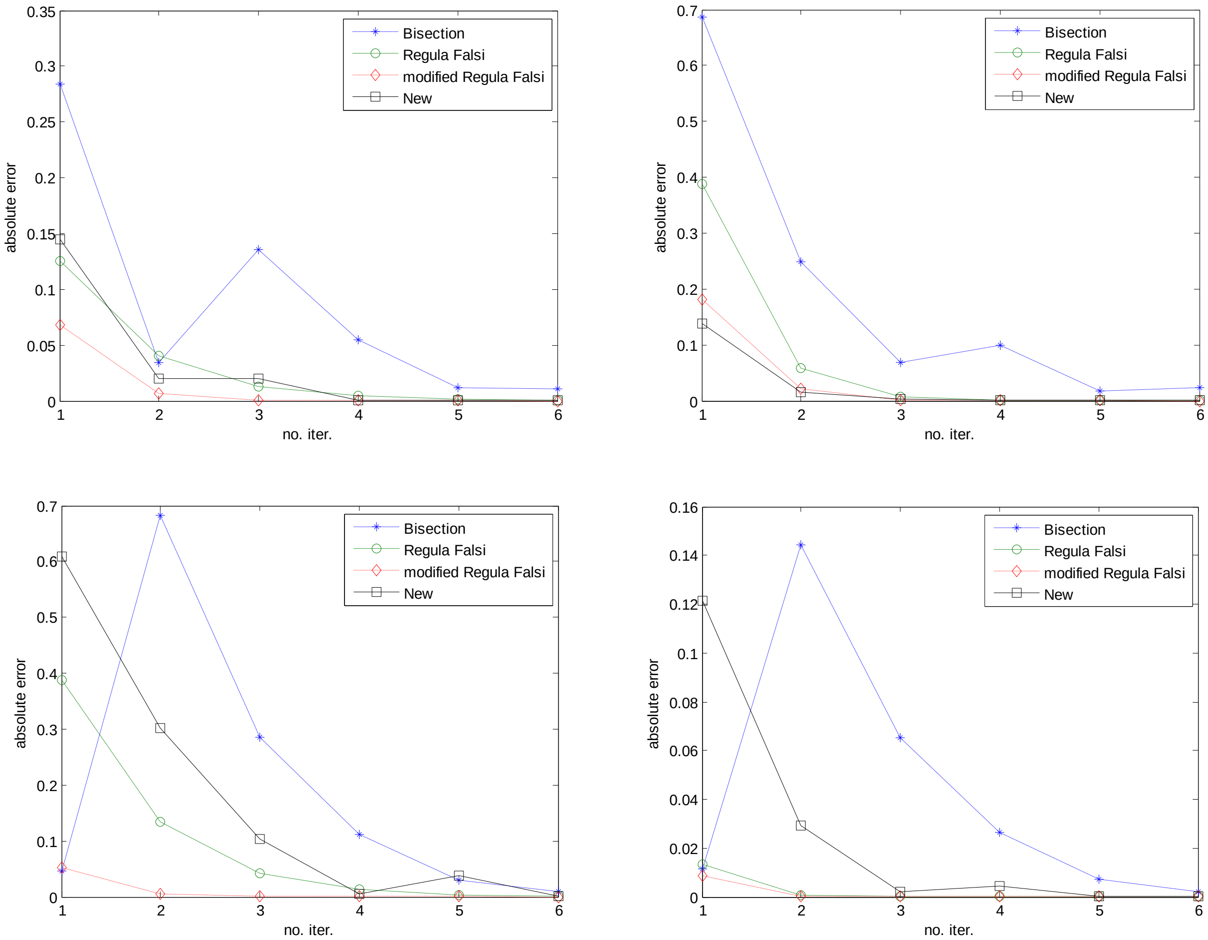

Figure 2: Rate of convergence of Bisection, Regula Falsi, modified Regula Falsi and new method of f_1 (top left), f_2 (top right), f_3 (bottom left) and f_4 (bottom right) on the first 6 iteration

5. CONCLUSION

The new algorithm for solving nonlinear equations has been presented. The algorithm feature is applied using nonlinear regression. Numerical comparisons indicate that this algorithm is much faster than BS and RF on both number of iteration and running time for the four examples. And the results are quite similar with MRF.

6. ACKNOWLEDGEMENT

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