Asian Journal of Applied Sciences (ISSN: 2321 – 0893)
Volume 03 – Issue 02, April 2015

Some Basic Properties of Γ-left-right Derivation in Γ-CI-Algebras

Pairote Yiarayong[1] and Phakakorn Panpho[2]

[1] Department of Mathematics, Faculty of Science and Technology,
Pibulsongkram Rajabhat University,Phitsanuloke 65000, Thailand
[2] Faculty of Science and Technology, Pibulsongkram Rajabhat University,
Phitsanuloke 65000, Thailand

Corresponding author's email: pairote0027 [AT] hotmail.com

ABSTRACT— ***The purpose of this paper is to introduce the notion of a*** Γ***-CI-algebras, we study*** Γ***-filters,*** Γ***-self-distributives,*** Γ***-transitives, upper sets and*** Γ***-left-right f-derivations in*** Γ***-CI-algebras. Some characterizations of*** Γ***-filters, upper sets and*** Γ***-left-right f-derivations are obtained. Moreover, we investigate relationships between*** Γ***-filters,*** Γ***-self-distributives and*** Γ***-transitives in*** Γ***-CI-algebras.***

Keywords— Γ-CI-algebra, Γ-filter, Γ-self-distributive, Γ-transitive, Γ-self-distributive

---

1. INTRODUCTION

In what follows, let X denote a CI-algebra unless otherwise specified. By a CI-algebra we mean an algebra $(X;*,1)$ of type (2, 0) with a single binary operation "$*$" that satisfies the following identities: for any $x, y, z \in X$,

1. $x*x=1$,
2. $1*x=x$,
3. $x*(y*z)=y*(x*z)$

In, 2006 , H. S. Kim and Y. H. Kim defined a BE-algebra [8]. Biao Long Meng, defined notion of CI–algebra as a generation of a BE–algebra BE-algebras and CI-algebras are studied in detail be some researchers (A. Borumand Saeid and A. Rezaei, 2012) and (B. Piekart and A. Andrzej Walendziak, 2011) some fundamental properties of CI–algebra are discussed.

In this paper is to introduce the notion of a Γ-CI-algebras, we study Γ-filters, Γ-self-distributives, Γ-transitives and upper sets in Γ-CI-algebras. Some characterizations of Γ-filters and upper sets are obtained. Moreover, we investigate relationships between Γ-filters, Γ-self-distributives and Γ-transitives in Γ-CI-algebras.

2. BASIC PROPERTIES

In this section we refer to [12] for some elementary aspects and quote few definitions and examples which are essential to step up this study. For more detail we refer to the papers in the references.

Definition 2.1. [12] Let X and Γ be any nonempty sets. If there exists a mapping $X\times\Gamma\times X\to X$ written as (x,γ,y) by $x\gamma y$, then $(\Gamma,X;1)$ is called a Γ-CI-algebra if

1. $x\gamma x = 1$ for all $x\in X$ and $\gamma\in\Gamma$,
2. $1\gamma x = x$ for all $x\in X$ and $\gamma\in\Gamma$,
3. $x\gamma(y\beta z) = y\gamma(x\beta z)$

for all $x,y,z\in X$ and $\gamma,\beta\in\Gamma$.

A Γ-CI-algebra $(\Gamma, X;1)$ is said to be Γ-self-distributive if $x\gamma(y\beta z) = (x\gamma y)\beta(x\gamma z)$ for all $x, y, z \in X$ and $\gamma, \beta \in \Gamma$.

Example 2.2. [12] Let $(\Gamma, X;1)$ be an arbitrary CI-algebra and Γ any nonempty set. Define a mapping $X \times \Gamma \times X \to X$, by $x\gamma y = xy$ for all $x, y \in X$ and $\gamma \in \Gamma$. It is easy to see that X is a Γ-CI-algebra. Indeed,

1. $x\gamma x = xx = 1$,
2. $1\gamma x = 1x = x$,
3. $x\gamma 1 = x1 = x$,
4. $x\gamma(y\beta z) = x(yz) = y(xz)$

for all $x, y, z \in X$ and $\gamma, \beta \in \Gamma$. Thus every CI-algebra implies a Γ-CI-algebra.

Example 2.3. [12] Let $X = \{1, a, b\}$ in which "$\cdot$" is defined by

$\cdot$	1	a	b
1	1	b	a
a	a	1	b
b	b	a	1

Then X is not a CI-algebra. Let $\Gamma \neq \varnothing$. Define a mapping $X \times \Gamma \times X \to X$ by $x\gamma y = y \cdot x$ for all $x, y \in X$ and $\gamma \in \Gamma$. Then X is a Γ-CI-algebra. Indeed,

1. $x\gamma x = (x \cdot x) \cdot (x \cdot x) = 1 \cdot 1 = 1$,
2. $1\gamma x = x \cdot 1 = x$,
3. $1\gamma x = x \cdot 1 = x$,
4. $x\gamma(y\beta z) = (y\beta z) \cdot x = (z \cdot y) \cdot x = (z \cdot x) \cdot y = (x\beta z) \cdot y = y\gamma(x\beta z)$

for all $x, y, z \in X$ and $\gamma, \beta \in \Gamma$. Therefore X is a Γ-CI-algebra.

Lemma 2.4. [12] Let X be a Γ-CI-algebra. Then $x\gamma y = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$.

Proposition 2.5. [12] Any Γ-CI-algebra X satisfies for any $x, y \in X$ and $\gamma, \beta, \alpha \in \Gamma$, $y\gamma\left[(y\alpha x)\beta x\right] = 1$

Proposition 2.6. [12] Any Γ-CI-algebra X satisfies for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, $(x\gamma 1)\beta(y\gamma 1) = (x\gamma y)\beta 1$.

Proposition 2.7. [12] Let X be a Γ-CI-algebra. If $x\gamma(x\beta y) = x\beta y$ for any $x, y \in X$ and $\gamma, \beta \in \Gamma$, then $x\gamma 1 = 1$.

3. Γ-FILTERS IN Γ-CI-ALGEBRAS

In this section, we study Γ-filters, Γ-self-distributives, Γ-transitives, Γ-self-distributive and upper sets in Γ-CI-algebras.

Definition 3.1. Let X be a Γ-CI-algebra and F a non-empty subset of X. Then F is said to be a Γ-filter of X if

1. $1 \in F$,
2. If $x \in F$ and $x\gamma y \in F$, then $y \in F$.

Proposition 3.2. If F_i, for all $i \in I$ are Γ-filters of X, then $\bigcap_{i \in I} F_i$ is a Γ-filter of Γ-CI-algebra X.

Proof. Straightforward.

Proposition 3.3. Let F be a subset of Γ-CI-algebra X. Then F is a Γ-filter of X if and only if for any $a, b \in F, \gamma, \beta \in \Gamma$ and $x \in X$, if $a\gamma(b\beta x) = 1$, then $x \in F$.

Proof. Let F be a Γ-filter of Γ-CI-algebra X. Assume that $a, b \in F, \gamma, \beta \in \Gamma$ and $x \in X$ such that a

$$a\gamma(b\beta x) = 1 \in F.$$

By Definition 3.1, we have $x \in F$. For the converse assume that for any $a, b \in F, \gamma, \beta \in \Gamma$ and $x \in X$, $a\gamma(b\beta x) = 1$ implies that $x \in F$. Suppose that a $x \in F$ and $x\gamma y \in F$. By Proposition 2.5, we have $a\gamma[(a\beta x)\alpha x] = 1$. Then $x \in F$ and hence F is a Γ-filter of X.

Definition 3.4. Let X be a Γ-CI-algebra and let $a \in X, \gamma \in \Gamma$. Define $A(a\gamma)$ by

$$A(a\gamma) = \{1\} \cup \{x \in X : a\gamma x = 1\}$$

Then we call $A(a\gamma)$ the initial section of the element a.

Lemma 3.5. Let X be a Γ-self distributive Γ-CI-algebra and $x\gamma y = 1$ for all $x, y \in X$, $\gamma \in \Gamma$. If $x \in A(a\gamma)$, then $y \in A(a\gamma)$.

Proof. Let X be a Γ- self distributive Γ-CI-algebra X and $x\gamma y = 1$ for all $x, y \in X, \gamma \in \Gamma$. Since $x \in A(a\gamma)$, we have $a\gamma x = 1$. Let $a \in A$ and let $x \in X$. Then

$$\begin{aligned} a\gamma y &= a\gamma(1\beta y) \\ &= a\gamma((x\gamma y)\beta y) \\ &= (a\gamma(x\gamma y))\beta(a\gamma y) \\ &= ((a\gamma x)\gamma(a\gamma y))\beta(a\gamma y) \\ &= (1\gamma(a\gamma y))\beta(a\gamma y) \\ &= (a\gamma y)\beta(a\gamma y) \end{aligned}$$

$$= \quad 1.$$

This implies $y \in A(a\gamma)$.

Lemma 3.6. Let X be a Γ-self distributive Γ-CI-algebra. Then $A(a\gamma)$ is a Γ-filter of X .

Proof. Let X be a Γ-self distributive Γ-CI-algebra. By Definition 3.4, we have $1 \in A(a\gamma)$. Let $x \in A(a\gamma)$ and $x\beta y \in A(a\gamma)$. Then $a\gamma x = 1$ and $a\gamma(x\beta y) = 1$ so that

$$\begin{aligned} a\gamma y &= 1\beta(a\gamma y) \\ &= (a\gamma x)\beta(a\gamma y) \\ &= a\gamma(x\beta y) \\ &= 1. \end{aligned}$$

This implies $y \in A(a\gamma)$ and hence $A(a\gamma)$ is a Γ-filter of X .

Proposition 3.7. Let X be a Γ-self-distributive Γ-CI-algebra and let $x, y, y \in X, \gamma, \beta \in \Gamma$. If $z\gamma(x\beta y) = 1$ and $z\gamma x = 1$, then $z\gamma y = 1$.

Proof. Let X be a Γ-self-distributive Γ-CI-algebra and $x, y, y \in X, \gamma, \beta \in \Gamma$. Suppose that $z\gamma(x\beta y) = 1$ and $z\gamma x = 1$. Then

$$\begin{aligned} z\gamma y &= 1\beta(z\gamma y) \\ &= (z\gamma x)\beta(z\gamma y) \\ &= z\gamma(x\beta y) \\ &= 1. \end{aligned}$$

Hence $z\gamma y = 1$.

Theorem 3.8. Let X be a Γ-CI-algebra, F a Γ-filter and $x \in F, \gamma \in \Gamma$. Then $A(x\gamma) \subseteq F$.

Proof. Let $y \in A(x\gamma)$. Then we have $x\gamma y = 1$. Since F is a Γ-filter of X and $x \in X$, we obtain $y \in F$. Therefore $A(x\gamma) \subseteq F$.

Definition 3.9. A Γ-CI-algebra X is said to be Γ-transitive if for all $x, y, z \in X$ and $\gamma, \alpha, \beta, \delta, \lambda \in \Gamma$, $(y\gamma z)\alpha((x\beta y)\delta(x\lambda z)) = 1$.

Proposition 3.10. If X is a Γ-self-distributive Γ-CI-algebra, then it is Γ-transitive.
Proof. For any $x, y \in X$ and $\gamma, \alpha, \beta, \delta, \lambda \in \Gamma$, we have

$$\begin{aligned} (y\gamma z)\alpha((x\beta y)\delta(x\lambda z)) &= (y\gamma z)\alpha((x\beta y)\delta(x\beta z)) \\ &= (y\gamma z)\alpha(x\beta(y\delta z)) \\ &= x\alpha((y\gamma z)\beta(y\delta z)) \\ &= x\alpha((y\delta z)\beta(y\delta z)) \\ &= x\alpha 1 \\ &= 1. \end{aligned}$$

Hence X is a Γ-transitive.

Definition 3.11. Let X be a Γ-CI-algebra and let $a,b \in X, \gamma, \beta \in \Gamma$. Define $A(a\gamma, b\beta)$ by

$$A(a\gamma, b\beta) = \{1\} \cup \{x \in X : a\gamma(b\beta x) = 1\}.$$

We call $A(a\gamma, b\beta)$ an upper set of a and b

Lemma 3.12. Let X be a Γ-CI-algebra and F be a Γ-filter of X. If $a\gamma 1 = 1$, then $1 \in F_{a\gamma} = \{x : a\gamma x \in F\}$, for any $x \in X$ and $\gamma \in \Gamma$.

Proof. Suppose that F is a Γ-filter of X. Since $1 = a\gamma 1$, we have $1 \in F_{a\gamma}$.

Theorem 4.13. Let X be a Γ-self distributive Γ-CI-algebra and F be a Γ-filter of X. If $a\gamma 1 = 1$, then $F_{a\gamma}$ is a Γ-filter, for any $x \in X$ and $\gamma \in \Gamma$.

Proof. Suppose that F is a Γ-filter of X. By Lemma 3.12, we have $1 \in F_{a\gamma}$. Assume $x \in F_{a\gamma}$ and $x\beta y \in F_{a\gamma}$. Then $a\gamma x \in F$ and $a\gamma(x\beta y) \in F$. We have

$$a\gamma(x\beta y) \quad = \quad (a\gamma x)\beta(a\gamma y)$$

so that $a\gamma y \in F$. Therefore $y \in F_{a\gamma}$ and hence $F_{a\gamma}$ is a Γ-filter of X.

Corollary 3.14. Let X be a Γ-CI-algebra and F be a Γ-filter of X. If $a\gamma 1 = 1$, then $A(a\gamma) \subseteq F_{a\gamma}$, for any $x \in X$ and $\gamma \in \Gamma$.

Proof. Suppose that F is a Γ-filter of X. Let $x \in A(a\gamma)$, for any $x \in X$ and $\gamma \in \Gamma$. Then $a\gamma x = 1 \in F$ so that $x \in F_{a\gamma}$. Hence $A(a\gamma) \subseteq F_{a\gamma}$.

4. Γ-LEFT-RIGHT DERIVATIONS IN Γ-CI-ALGEBRAS

In this section, we introduce a relation "$\leq$" on X by $x \leq y$ if and only if $x\gamma y = 1$ for all $\gamma \in \Gamma$.

Remark Let X be a Γ-CI-algebra X, $a,b,x \in X$ and let $\gamma, \beta \in \Gamma$. Then

1. $y \leq [(y\alpha x)\beta x]$
2. $x \in A(a\gamma) \quad \Leftrightarrow \quad a \leq x$
3. $x \in A(a\gamma, b\beta) \quad \Leftrightarrow \quad a \leq (b\beta x)$.

Lemma 4.1. Let X be a Γ-CI-algebra and let $x \in X$. If $1 \leq x$, then $x = 1$.

Proof. Suppose that X is a Γ-CI-algebra. Since $1 \leq x$, we have $1\gamma x = 1$. By Definition 2.1, we have $1\gamma x = x$. Then $x = 1$.

Lemma 4.2. Let X be a Γ-CI-algebra. If X is a Γ-transitive, then for all $x, y, z \in X, \gamma, \beta \in \Gamma$, then $y \leq z$ implies that $x\beta y \leq x\gamma z$.

Proof. Suppose that X is a Γ-transitive, $x, y, z \in X$ and let $\gamma, \beta, \alpha \in \Gamma$. Since $x \leq y$, we have $x\alpha y = 1$. Then

$$(x\beta y)\lambda(x\gamma z) \; = \; (x\beta y)\lambda(x\gamma z)$$

$$= 1\delta\left[(x\beta y)\lambda(x\gamma z)\right]$$
$$= (y\alpha z)\delta\left[(x\beta y)\lambda(x\gamma z)\right]$$
$$= 1.$$

Hence $x\beta y \le x\gamma z$.

Proposition 4.3. Let X be a Γ-self distributive Γ-CI-algebra. For all $x, y, z \in X$ and $\gamma, \beta, \alpha \in \Gamma$ if $x\gamma 1 = 1$, then

1. if $x \le y$, then $z\gamma x \le z\gamma y$,
2. $y\gamma z \le (x\beta y)\gamma(x\alpha z)$.

Proof. Let X be a Γ-self distributive Γ-CI-algebra, $x, y, z \in X$ and $\gamma, \beta, \alpha \in \Gamma$.

1. Suppose that $x \le y$. Then $x\beta y = 1$. We have

$$\begin{aligned}(z\gamma x)\alpha(z\gamma y) &= z\gamma(x\alpha y)\\ &= z\gamma(x\beta y)\\ &= z\gamma(x\gamma y)\\ &= z\gamma 1\\ &= 1.\end{aligned}$$

Hence $z\gamma x \le z\gamma y$.

2. Now consider

$$\begin{aligned}(y\gamma z)\delta[(x\beta y)\gamma(x\alpha z)] &= (y\gamma z)\delta[(x\beta y)\gamma(x\beta z)]\\ &= (y\gamma z)\delta[x\beta(y\gamma z)]\\ &= x\delta[(y\gamma z)\beta(y\gamma z)]\\ &= x\delta 1\\ &= 1.\end{aligned}$$

Hence $y\gamma z \le (x\beta y)\gamma(x\alpha z)$.

Proposition 4.4. Let X be a Γ-CI-algebra and F be a Γ-filter of X. Then for all $x, y \in X$ and $\gamma \in \Gamma$ the following statements hold:

1. If $x \in F$ and $x \le y$, then $y \in F$.
2. If X is a Γ-self distributive Γ-CI-algebra and $x\beta y = 1$ for all $x, y \in F$ and $\gamma, \beta \in \Gamma$, then $x\gamma y \in F$.

Proof. 1. Suppose that X is a Γ-CI-algebra and F is a Γ-filter of X. Let $x \in F$ and $x \le y$. Then $x\gamma y = 1$ so that $x\gamma y \in F$. Therefore $y \in F$.

2. Suppose that X is a Γ-self distributive Γ-CI-algebra and $x\beta y = 1$ for all $x, y \in F$ and $\gamma, \beta, \alpha \in \Gamma$. We have

$$\begin{aligned}y\gamma(x\beta(x\alpha y)) &= x\gamma(y\beta(x\alpha y))\\ &= x\gamma((y\beta x)\alpha(y\beta y))\\ &= x\gamma((y\beta x)\alpha 1)\\ &= [x\gamma(y\beta x)]\alpha(x\alpha 1)\\ &= [(x\gamma y)\beta(x\beta x)]\,\alpha(x\alpha 1)\\ &= [(x\gamma y)\beta 1]\alpha(x\alpha 1)\\ &= [(x\beta y)\beta 1]\alpha(x\alpha 1)\end{aligned}$$

$$\begin{aligned} &= (1\beta 1)\alpha(x\alpha 1) \\ &= 1\alpha(x\gamma 1) \\ &= x\gamma 1 \\ &= 1 \end{aligned}$$

and hence $y\gamma(x\beta(x\alpha y)) \in F$. Since $y \in F$, we have $x\beta(x\alpha y) \in F$. It follows $x\gamma y = x\alpha y \in F$. For elements x and y of a Γ-CI-algebra X, denote $x \wedge y = (y\gamma x)\gamma x$.

Definition 4.5. Let X be a Γ-CI-algebra. A mapping $d : X \to X$ is a Γ-left-right derivation (briefly, Γ-(l, r)-derivation) of X, if it satisfies the identity $d(x\gamma y) = d(x)\gamma y \wedge x\gamma d(y)$ for all $x, y \in X$ and $\gamma \in \Gamma$. If d satisfies the identity $d(x\gamma y) = x\gamma d(y) \wedge d(x)\gamma y$ for all $x, y \in X$ and $\gamma \in \Gamma$, then d is a Γ-right-left derivation (briefly, Γ-(r, l)-derivation) of X. Moreover, if d is both a Γ-(l, r) and Γ-(r, l)-derivation, then d is a Γ-derivation of X.

Proposition 4.6. If d is a Γ-(l,r)-derivation of Γ-self-distributive X, then, $d(1) = 1$.

Proof. Let X be a Γ-CI-algebra and let d be a Γ-left-right derivation of X. Then

$$\begin{aligned} d(1) &= d(1\gamma 1) \\ &= d(1)\gamma 1 \wedge 1\gamma d(1) \\ &= d(1)\gamma 1 \wedge d(1) \\ &= [d(1)\beta(d(1)\gamma 1)]\beta(d(1)\gamma 1) \\ &= [(d(1)\beta d(1))\gamma(d(1)\beta 1)]\ \beta(d(1)\gamma 1) \\ &= [1\gamma(d(1)\beta 1)]\ \beta(d(1)\gamma 1) \\ &= (d(1)\beta 1)\ \beta(d(1)\gamma 1) \\ &= (d(1)\gamma 1)\ \beta(d(1)\gamma 1) \\ &= 1. \end{aligned}$$

Hence $d(1) = 1$.

Definition 4.7. A Γ-derivation d of a Γ-CI-algebra X is said to be Γ-regular if $d(1) = 1$.

Corollary 4.8. A Γ-(1,r)-derivation d of a Γ-self-distributive X is Γ-regular.
Proof. Straightforward. □

Proposition 4.9. Let d be a Γ-(1,r)-derivation of a Γ-self-distributive X. Then $d(x) = x \wedge d(x)$ for all $x \in X$.

Proof. Let d be a Γ-(1,r)-derivation of a Γ-self-distributive X and $x \in X, \gamma \in \Gamma$. Then

$$\begin{aligned} d(x) &= d(1\gamma x) \\ &= d(1)\gamma x \wedge 1\gamma d(x) \\ &= d(1)\gamma x \wedge d(x) \\ &= 1\gamma x \wedge d(x) \end{aligned}$$

$$= \quad x \wedge d(x).$$

Hence $d(x) = x \wedge d(x)$.

Definition 4.10. Let X a Γ-CI-algebra. A map $f : X \to X$ is called endomorphism if $f(x\gamma y) = f(x)\gamma f(y)$ for all $x, y \in X$ and $\gamma \in \Gamma$.

Definition 4.11. Let f be an endomorphism of a Γ-CI-algebra X. A map $d_f : X \to X$ satisfying the identity

$$d_f(x\gamma y) = d_f(x)\gamma f(y) \wedge f(x)\gamma d_f(y)$$

is called a Γ-left-right f-derivation (briefly, Γ-(l; r)-f-derivation) of X.

Proposition 4.12. Let d_f be a Γ-(1,r)-derivation of a Γ-self-distributive X. Then $d_f(x) = 1$ for all $x \in X$.

Proof. Let d_f be a Γ-(1,r)-derivation of a Γ-self-distributive X. For all $\gamma, \beta \in \Gamma$ so that

$$\begin{aligned}
d_f(1) &= d_f(1\gamma 1) \\
&= d_f(1)\gamma f(1) \wedge f(1)\gamma d_f(1) \\
&= d_f(1)\gamma f(1) \wedge f(1)\gamma d_f(1) \\
&= d_f(1)\gamma f(1\gamma 1) \wedge f(1\gamma 1)\gamma d_f(1) \\
&= d_f(1)\gamma[f(1)\gamma f(1)] \wedge [f(1)\gamma f(1)]\gamma d_f(1) \\
&= d_f(1)\gamma 1 \wedge 1\gamma d_f(1) \\
&= d_f(1)\gamma 1 \wedge d_f(1) \\
&= [d_f(1)\beta(d_f(1)\gamma 1)]\beta\, (d_f(1)\gamma 1) \\
&= [(d_f(1)\beta d_f(1))\gamma(d_f(1)\beta 1)]\ \beta(d_f(1)\gamma 1) \\
&= [1\gamma(d_f(1)\beta 1)]\ \beta(d_f(1)\gamma 1) \\
&= (d_f(1)\beta 1)\ \beta(d_f(1)\gamma 1) \\
&= (d_f(1)\gamma 1)\ \beta(d_f(1)\gamma 1) \\
&= 1.
\end{aligned}$$

Hence $d_f(1) = 1$

Proposition 4.13. Let d_f be a Γ-(1,r)-derivation of a Γ-self-distributive X. Then $d_f(x) = d_f(x) \wedge f(x)$ for all $x \in X$.

Proof. Let d_f be a Γ-(1,r)-derivation of a Γ-self-distributive X. For all $\gamma, \beta \in \Gamma$ so that

$$\begin{aligned}
d_f(x) &= d_f(1\gamma x) \\
&= d_f(1)\gamma f(x) \wedge f(1)\gamma d_f(x) \\
&= 1\gamma f(x) \wedge 1\gamma d_f(x) \\
&= f(x) \wedge d_f(x).
\end{aligned}$$

Hence $d_f(x) = d_f(x) \wedge f(x)$.

Theorem 4.14. If d_f is a Γ-regular Γ-(l; r)-f-derivation of a Γ-CI-algebra X, then $d_f(x) = f(x) \wedge d_f(x)$

Proof. Let d_f be a Γ-regular Γ-(l; r)-f-derivation of a Γ-CI-algebra X. For all $\gamma, \beta \in \Gamma$ so that

$$\begin{aligned} d_f(x) &= d_f(1\gamma x) \\ &= d_f(1)\gamma f(x) \wedge f(1)\gamma d_f(x) \\ &= 1\gamma f(x) \wedge 1\gamma d_f(x) \\ &= f(x) \wedge d_f(x). \end{aligned}$$

Hence $d_f(x) = f(x) \wedge d_f(x)$.

5. ACKNOWLEDGEMENT

The authors are very grateful to the anonymous referee for stimulating comments and improving presentation of the paper.

6. REFERENCES

[1] Ahn S.S, Kim Y.H., So K.S., "Fuzzy BE-algebras. Journal of applied mathematics and informatics", vol. 29, 1049-1057, 2011.

[2] Ahn S.S., So Y.H., "On ideals and upper sets in BE-algebras", Sci. Math. Jpn. Online e-2008, vol. 2, 279-285, 2008.

[3] Borumand Saeid A., Rezaei A.,"Quotient CI–algebras", Bulletin of the Transilvania University of Bra¸sov, vol. 5, no. 54, 15-22, 2012.

[4] Hu Q.P., Li X., "On BCH-algebras", Math.Seminar Notes, vol. 11, 313-320, 1983.

[5] Hu Q. P., Li X., "On proper BCH-algebras", Math Japonica, vol. 30, 659-661, 1985.

[6] Iseki K., Tanaka S., "An introduction to theory of BCK-algebras", Math Japonica, vol. 23, 1-20, 1978.

[7] Kim K.H., "A Note on CI-algebras", International Mathematical Forum, vol. 6, no. 1, 1-5, 2011.

[8] Kim H.S., Kim Y.H., "On BE-Algebras", Sci. Math. Jpn., vol. 66, no. 1, 1299-1302, 2006.

[9] Meng B.L., "CI–algebra", Sci. Math. Jpn. vol. 71, no. 2, 695-701, 2010.

[10] Neggers J., Ahn S.S., Kim H.S., "On q-algebras", Int. J. Math. Math. Sci., vol. 27, no. 12, 749-757, 2001.

[11] Piekart B., Andrzej Walendziak A., "On filters and upper sets in CI-algebras", Algebra and Discrete Mathematics, vol. 11, no. 1, 109-115, 2011.

[12] Yiarayong P., and Panpho P., "On Γ-CI-Algebras", Asian Journal of Applied Sciences, vol. 2, no. 6, 952-956, 2004.