Thermeal Diffusion and Diffusion Thermo Effects on the Motion of Viscoelastic Fluid through Porous Medium with Chemical Reaction under the Influence of Uniform Magnetic Field

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ABSTRACT--- *The motion of non-Newtonian fluid through porous medium inside the boundary layer of an infinite wall is considered. The fluid is obeying the viscoelastic type and the motion is under the effects of magnetic field with chemical reaction and heat generation. The problem is modulated mathematically by using the continuity, momentum, heat and mass transfer equations. The system of differential equations which describe this motion is solved numerically with appropriate boundary conditions by using finite difference scheme. Numerical results are presented to investigate the influence of the magnetic parameter, visco-elastic parameter, porous medium parameter, Prandtl number, heat source sink parameter, Dufour number, Hartmann number, Eckert number, Sort number, Schmidt number and the chemical reaction parameter on the stream function, temperature and concentration profiles. Several graphs for these results of physical interest are displayed and discussed in detail.*

Keywords--- Viscoelasic Fluid ; Magnetic Field; Porous Medium; Thermal Diffusion; Diffusion Thermo; Chemical Reaction.
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1- INTRODUCTION

The study of visco-elastic fluids has gained great importance in the last few years . Visoelastic flows arise in numerous processes in chemical engineering system. Such flows possess both viscous and elastic properties. Veena et al. [1] investigated the study of non-similar solutions for an electrically conducting viscoelastic fluid flow with heat and mass transfer over a stretching sheet embedded in saturated porous medium. Eldabe and Sallam [2] studied the steady flow of an electrically conducting viscoelastic fluid in a saturated porous medium between two porous parallel plates with one plate moving and the other stationary. Also, they [3] considered the problem of the flow and heat transfer in the non-Newtonian viscoelastic electrically conducting incompressible fluid confined between two parallel plates one of them is a stretching plate and the other is a stationary porous plate through which there is a uniform injection. The effects of chemical reaction and heat radiation on the MHD flow of viscoelastic fluid through a porous medium over a horizontal stretching flat plate is studied by Eldabe et al. [4] . Alharbi et al [5] investigated the problem of convective heat and mass transfer of incompressible MHD viscoelastic fluid embedded in a porous medium over a stretching sheet under a chemical reaction. The effect of thermal diffusion of unsteady flow of a viscoelastic fluid through porous medium with heat source is investigated by Kumar et al. [6] in presence of a uniform magnetic field. Hayat et al .[7] discussed the characteristics of hydrodynamic flow and heat transfer analysis in a rotating frame of reference. An analysis was carried out by Abel and Joshi [8] to study the effect of heat transfer in MHD viscoelstic fluid over a non- isothermal stretching sheet with internal heat generation.

The heat and mass transfer simultaneously affecting each other that will cause the cross diffusion effect. The heat transfer caused by concentration gradient is called the diffusion- thermo or Dufour effect. On the other hand mass transfer caused by temperature gradient is called Soret or thermo-diffusion effect. Alam and Rahman [10] investigated Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. El-Arabawy [11] investigated the heat and mass transfer by natural convection from vertical surface embedded in a fluid-saturated porous

media considering Soret and Dufour effects with variable surface temperature and constant concentration. The thermal-diffusion and diffusion-thermo effects on mixed free-forced convection and mass transfer boundary layer flow of non-Newtonian fluid with temperature dependent viscosity are studied by Eldabe et al [12]. Also, Eldabe and Abu Zeid [13] investigated thermal diffusion and diffusion thermo effects on the viscous fluid flow with heat and mass transfer through porous medium over a shrinking sheet. The problem of non-Newtonian magnetohydrodynamic flow over a stretching surface with heat and mass transfer was studied by Abel et al [14].

The main aim of this work is to extend the work of Abel et al [14] to include the motion through porous medium in the presence of chemical reaction and considering thermal diffusion and diffusion thermo effects with Joule dissipations and viscous. The governing non-linear partial differential equations are transformed by using similarity transform into non-linear ordinary differential equations. The system of these equations with an appropriate boundary conditions are solved numerically by using the finite different method. The effects of various governing parameter on the stream function, temperature and concentration are shown in figures and discussed in details.

NOMENCLATURE

A_0, A_1 constants

B_0 Strength of the magnetic field

C Species concentration in the fluid

c_p Specific heat at constant pressure

C_∞ Ambient concentration of the fluid

C_w The wall concentration of fluid

D Mass diffusion coefficient

Df Dufour number $= \dfrac{D_m k_T (C_\varpi - C_\infty)}{\upsilon c_s c_p (T_\varpi - T_\infty)}$

Ec Eckert number $= \dfrac{\lambda^2 l^2}{A_0 c_p}$

K The permeability parameter $= \dfrac{\mu}{\rho k_p \lambda}$

K_1 The elasticity parameter $= \dfrac{k_0 \lambda}{\upsilon}$

L The chemical reaction parameter $= \dfrac{k}{\lambda}$

l The characteristic length

M The magnetic parameter $= \dfrac{\sigma B_0^2}{\lambda \rho}$

Pr prandtl number $= \dfrac{\mu c_p}{\alpha}$

p The pressure

Sc Shmidt number $= \dfrac{\upsilon}{D_m}$

Sr Sort number $= \dfrac{D_m k_T (T_\varpi - T_\infty)}{\upsilon T_m (C_\varpi - C_\infty)}$

t The transpose

Ha Hartmann number $= \sqrt{\dfrac{\sigma}{\rho}} \, B_0$

T temperature of the fluid

T_∞ Ambient temperature of the fluid

T_w wall temperature of the fluid

v transverse velocity in thy directions

Greek symbol

α Coefficient of thermal conductivity

α_1, α_2 are the normal stress modulus

$\beta = \dfrac{Q}{\rho \lambda c_p}$ The heat source sink

ρ Density of fluid, electrical

λ Constant

μ The coefficient of viscosity

υ The kinematic of viscosity

σ^* Stefan- Boltzman constant parameter

τ The stress tensor

τ_{xy} Component of the stress tensor

2- FLOW ANALYSIS

Consider the two- dimensional laminar boundary layer flow of an incompressible viscoelastic fluid through porous medium with heat and mass transfer past an infinite plate.

The system is stressed by the influence of magnetic field in the presence of chemical reaction and heat generation. Choose the Cartesian coordinates (x, y) where x - axis is along the infinite plate while y - axis is perpendicular to it.

The constitutive equation of the viscoelastic fluid which illustrates the relation between stress and rate of strain can be written as

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

Here A_1 and A_2 are defined as

$$A_1 = (\text{grad } \nu) + (\text{grad } \nu)^t \tag{2}$$

$$A_2 = (\frac{d}{dt} A_1) + A_1 . (\text{grad } \nu)^t + (\text{grad } \nu)^t . A_1 \tag{3}$$

The thermodynamics and stability of the model in Eq. (1) were studied in detail by Dunn and Fosdick [9]. Thermodynamics compatibility in the sense that all motions of the fluid meet the Clausius-Duhem inequality (generally interpreted as a statement of the second law of thermodynamics) and the assumption that the specific Helmholtz free energy of the fluid be a minimum when the fluid is locally at rest requires that

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad \alpha_1 + \alpha_2 = 0 \tag{4}$$

In our analysis we assume that the fluid is thermodynamically compatible; hence the stress constitutive relation, Eq. (1) reduces to

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 - \alpha_1 A_1^2 \tag{5}$$

3-GOVERNING EQUATIONS

In Cartesian coordinates the continuity, momentum, temperature and concentration equations can be written as
The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

The momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} - (\frac{\mu}{\rho k_p} + \frac{\sigma B_0^2}{\rho}) u - \frac{k_0}{\rho} \left\{ \frac{\partial}{\partial x} (u \frac{\partial^2 u}{\partial y^2}) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right\} \tag{7}$$

The energy equation (heat transfer) is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p} u^2 - k_0 \frac{\partial u}{\partial y} \left\{ \frac{\partial}{\partial y} (u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) \right\} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + Q(T - T_\infty) \tag{8}$$

The diffusion equation (mass transfer) is

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + k_1 C + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{9}$$

The appropriate boundary conditions governing the problem are :

$$\left. \begin{array}{l} u = u_w = c x \ , v = 0, T = T_w = T_\infty + A_0 \left(\frac{x}{l} \right)^2, C = C_w = C_\infty + A_1 \left(\frac{x}{l} \right) \quad \text{at } y = 0 \\[2mm] u \rightarrow 0, v \rightarrow 0, \ T \rightarrow T_\infty, C \rightarrow C_\infty \qquad \text{as } y \rightarrow \infty \end{array} \right\} \tag{10}$$

The equation of continuity is satisfied if we choose a dimensionless stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \qquad (11)$$

Introducing the similarity transformations

$$\eta = \sqrt{\frac{\lambda}{\upsilon}}\, y \qquad f(\eta) = -\frac{\psi(x, y)}{x\sqrt{\upsilon\lambda}} \qquad (12)$$

Using Eqs. (11) and (12), the velocity components become

$$u = \lambda x f'(\eta)\,, \quad v = -\sqrt{\upsilon\lambda}\, f(\eta) \quad \text{and also } \theta = \frac{T - T_\infty}{T_w - T_\infty}\,, \quad \varphi = \frac{C - C_\infty}{C_w - C_\infty}\,, \quad \upsilon = \frac{\mu}{\rho}\,. \qquad (13)$$

Where a prime denotes differentiation with respect to η.

Substitution from Eq. (13) in Eqs. (7-9) we get

$$\left.\begin{array}{l} f'^2 - f f'' = f''' - (K + M)f' - K_1(2f'f''' - f''^2 - f f^{(4)}) \\[6pt] \Pr(2f'\theta - \theta'f) = \theta'' + Ec\,\Pr(Ha)^2 f'^2 + Df\,\Pr\varphi'' + \beta\,\Pr\theta - K_1 Ec\,(f'f''^2 - f f''f''') \\[6pt] (2f'\varphi - f\varphi')Sc = \varphi'' + Sc\,(Sr\,\theta'' + L\,\varphi) \end{array}\right\} \qquad (14)$$

subject to the following

$$\left.\begin{array}{l} f' = 1,\ f = 0, \theta = 1, \varphi = 1 \qquad \text{with } \eta \to 0 \\[6pt] f' = 0,\ f = 0, \theta = 0, \varphi = 0 \qquad \text{as } \quad \eta \to \infty \end{array}\right\} \qquad (15)$$

The system of coupled equations (14), subjected to the boundary conditions (15), is solved numerically by using the finite different scheme in the following section.

4- NUMERICAL RESULTS AND DISCUSSION

The system of equations that govern the motion with heat and mass transfer of magnetohydrodynamic are solved numerically by using the explicit finite difference scheme. The numerical formulas for stream function, temperature and concentration are obtained as a functions of the physical parameters of the problem.

The effects of physical parameters on the stream function ψ are indicated through figures (1-3). Figures (1) and (2), illustrate the effects of the permeabilty parameter K and the elasticity K_1 respectively. It is found that the stream function ψ increases with increasing K, but it decreases with increasing K_1. we also noted that the stream function ψ increases till a definite maximum value at $\eta = \eta_0$ and it decreases afterwards. This maximum value of ψ increases by increasing K while it decreases by increasing K_1.

The effect of the magnetic parameter M on the stream function ψ is shown in figure (3), and it is shown that the stream function ψ increases by increasing M, and also noted that the stream function ψ increases till a definite maximum value at $\eta = \eta_0$ and it decreases afterwards.

The temperature distribution θ for different values of Eckert number Ec is obtained graphically through figure (4), we observed that the temperature distribution θ decreases with the increasing of Ec in the region $0 \prec \eta \prec 3$. Afterwards θ increases with increasing Ec, and the obtained curves intersect approximately at $\eta = 3$. It is also noted that θ decreases with the increase of x till a definite value $\eta = \eta_0$ (represents the minimum value of θ) and it increases afterwards till a maximum value of θ, then it decreases again. In Figs. (5-8), the effects of the permeabilty K, the elasicity parameter K_1, the magnetic field M and Prandtl number $\Pr$ on the temperature distribution θ respectively. In these figures, we observed that the effects both of K, K_1, M and $\Pr$ on θ are similar to the effect of Ec on θ with the only different that the obtained curves are very close to those obtained in Fig. (4).

The effect of Dufour number Df on the temperature distribution θ is shown in figure (9), and it is shown that the temperature θ increases by increasing of Df. The change of the temperature distribution θ with for different values of the Schemidt number Sc shown through Fig. (10), it is clear that the temperature distribution increases by increasing of

Sc . Figs. (11) and (12) illustrate that the behaviors of the temperature distribution θ with different values of the chemical reaction L and the Hartmann number Ha respectively. It is found that θ increases with increasing L , while it decreases with increasing Ha . We noted that the curves coincide in the region $0 \prec \eta \prec 1$, afterwards increases by increasing L but decreases by decreasing of Ha . The temperature distribution θ for different values of Sr is displayed in Fig. (13). The graphical results of fig. (13) indicate that the temperature θ increases as Sr increases. It is obvious that the curves of θ are coincide in the region $0 \prec \eta \prec 2$ and afterwords the temperature θ increases by increasing of Sr . In figure (14), the graphs of the temperature distribution θ have been drawn for different values of the heat source sink β . It is seen that, the behavior of the curves are coincide in the region $0 \prec \eta \prec 3$ and afterwords θ decreases by increasing of β .

Figure (15) and (16), show that the behaviors of the concentration distribution C for different values of permeabilty parameter K and the elasticity parameter K_1 respectively. It is found that the concentration C increases with increasing the porosity parameter K , whereas it decreases with increasing elasticity parameter K_1 . We also noted that the curves are coincide in the region $0 \prec \eta \prec 1$ and afterwords, the concentration C increases by increasing K and the concentration C decreases by increasing of K_1 . The effects of Schmidt number Sc on the concentration distribution C is shown in figure (17), and it is shown that the concentration C increases by increasing Sc . It is observed also that the behavior of the curves are coincide in the region $0 \prec \eta \prec 1$ and afterwords, the concentration C increases by increasing of Sc .

The effects of the magnetic field parameter M and the chemical reaction parameter L on the concentration distribution are shown in figures (18) and (19). It is seen the behavior of the curves are the same as that obtained in Fig. (17), except that the obtained curves are close to those obtained in Fig. (17).

Finally, Fig. (20), illustrates the changes of the concentration C with Soret number Sr . It is clear that the concentration C increases with increasing of Sr .

5- CONCLUSION

. This problem is modulated mathematically by a set of differential equations which discribed the motion of the viscoelastic fluid with heat and mass transfer . These equations are transformed into an ordinary differential equations by using a similarty method which are solved numerically by using explicit finite difference scheme with appropriate boundary conditions. The effects of the parameters of the problem on theses solutions are discussed and observed through several figures. The very important points conclusion can be written :

a. The stream function ψ increases with the increase in magnetic parameter M and porosity parameter K increases.

b. The stream function ψ decreases when elasticity parameter K_1 increase.

c. The temperature θ increases when the Prantl number Pr , Dufour number Df , the Schmidt number Sc , porosity parameter K , elasticity parameter K_1 , the magnetic field parameter M , the chemical reaction parameter L , the Soret number Sr and the Eckert number Ec increase.

d. The temperature decreases when the heat source sink β and the Hartmann number Ha decrease.

e. The concentration C increases when the Schmidt number Sc , the porosity parameter K , the magnetic parameter M , the chemical reaction parameter L , and Soret number Sr increase.

f. We noted that the concentration C and The stream function ψ decrease when the function ψ decrease when the elasticity parameter K_1 increases.
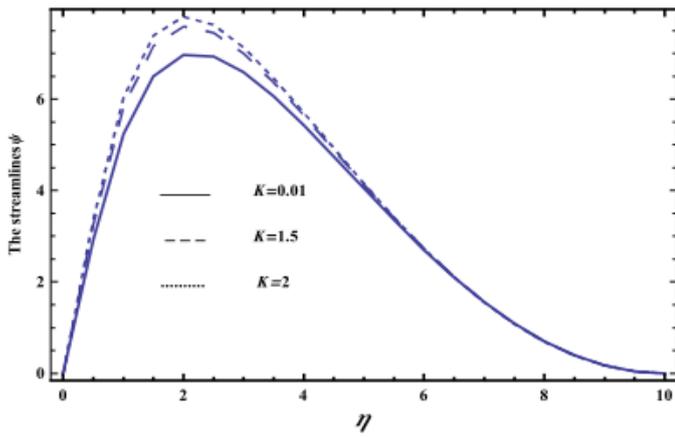
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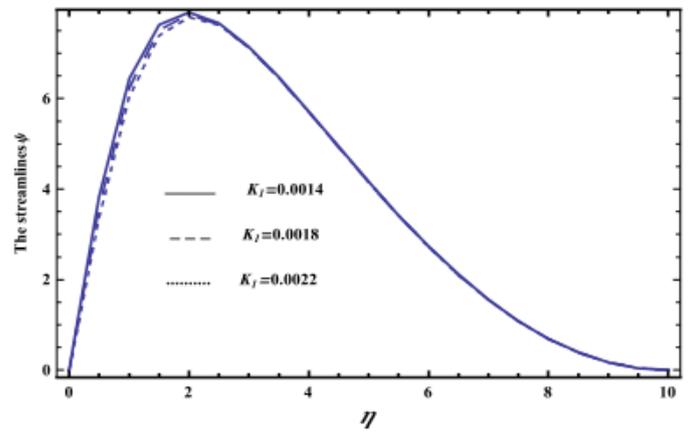
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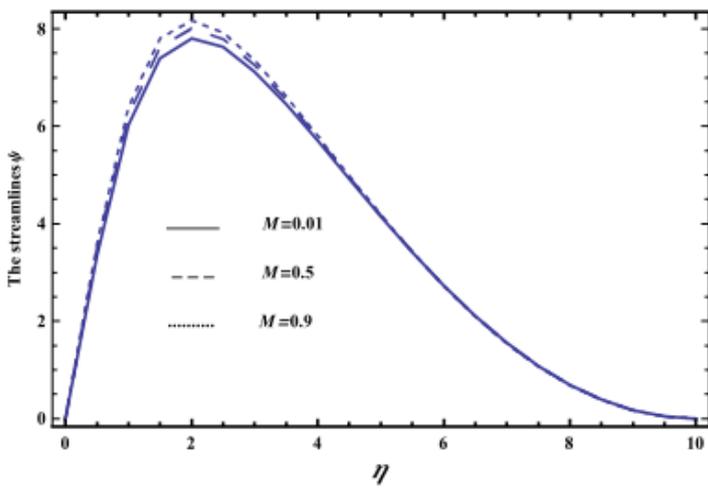
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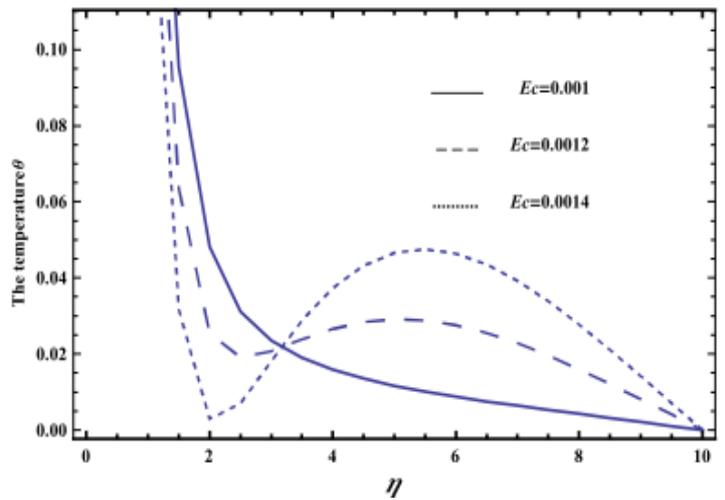
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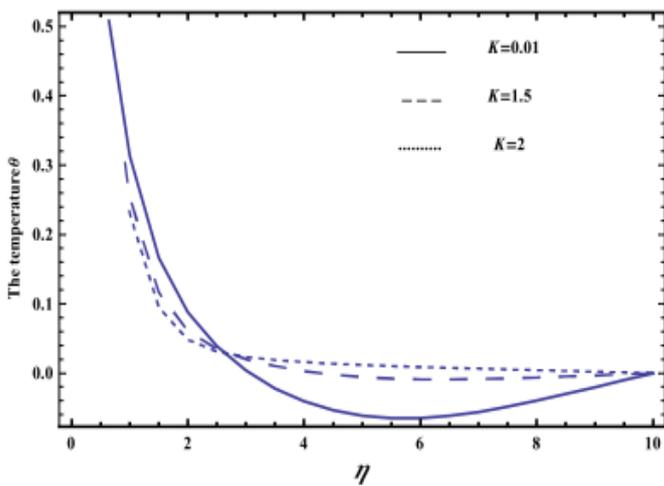
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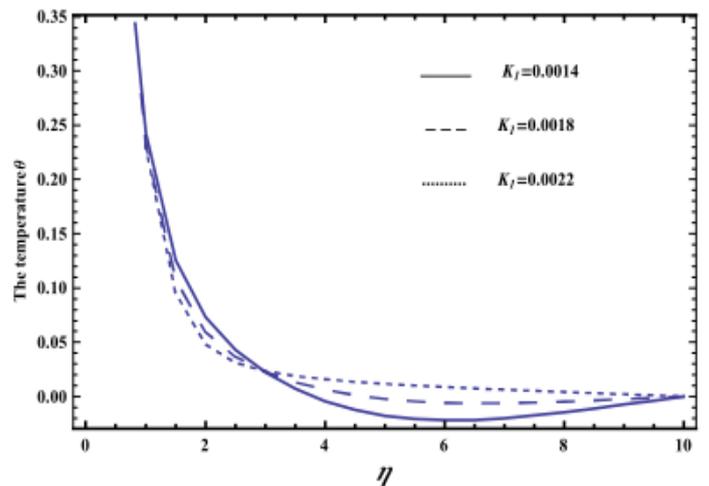
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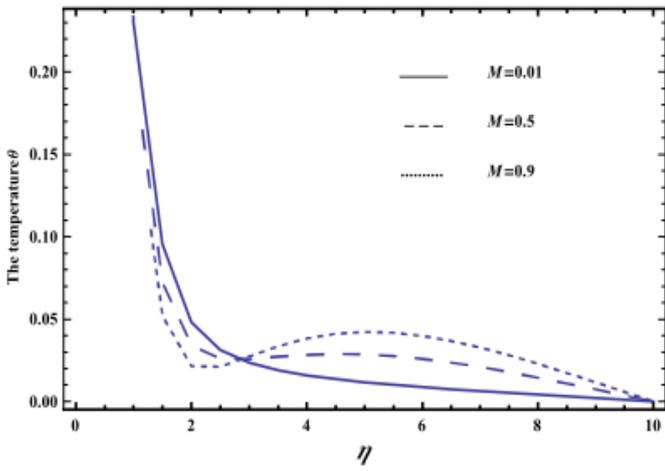
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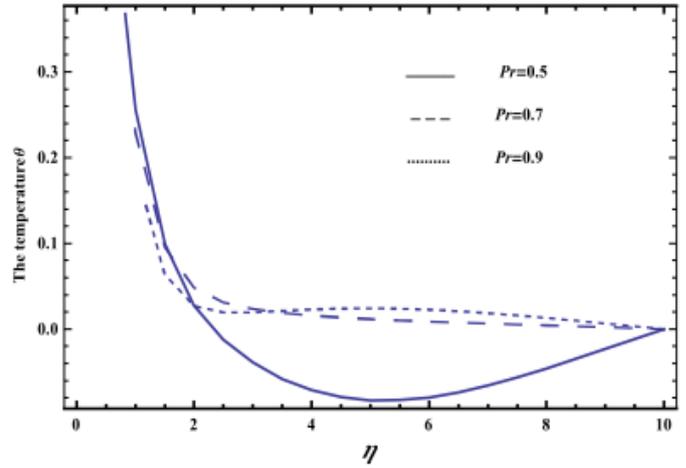
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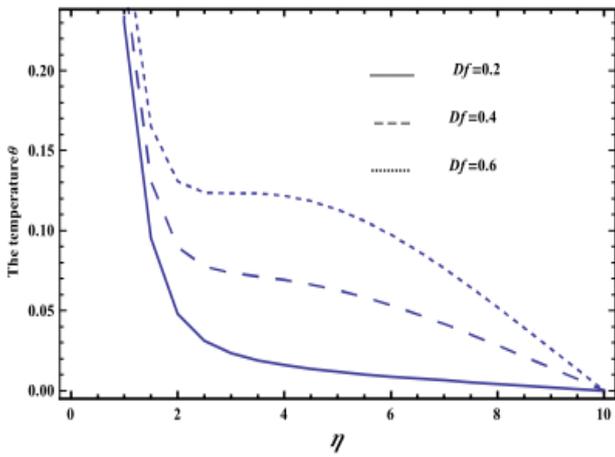
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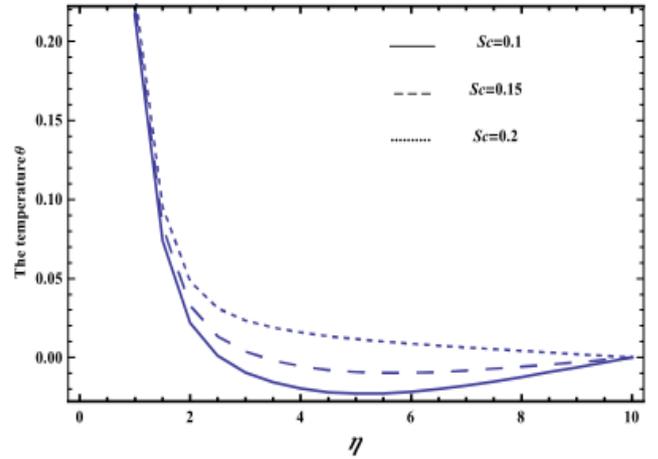
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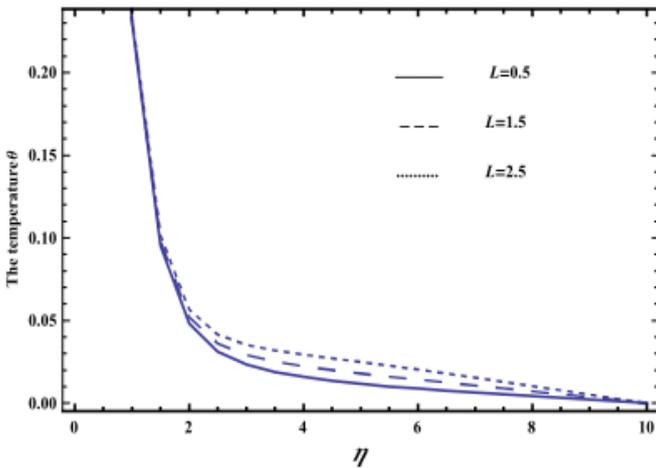
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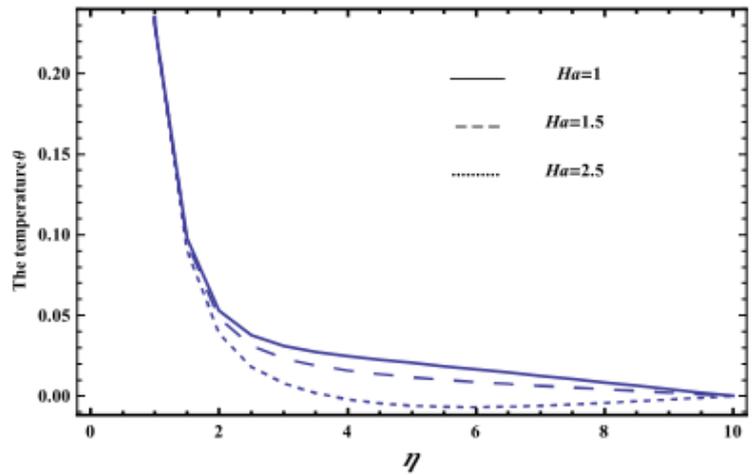
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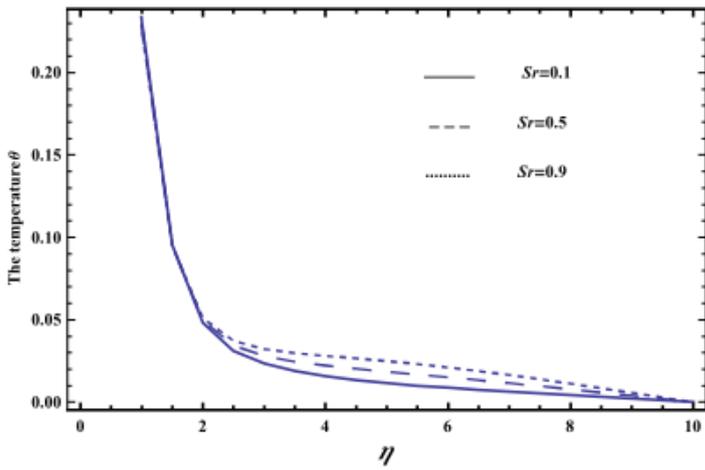
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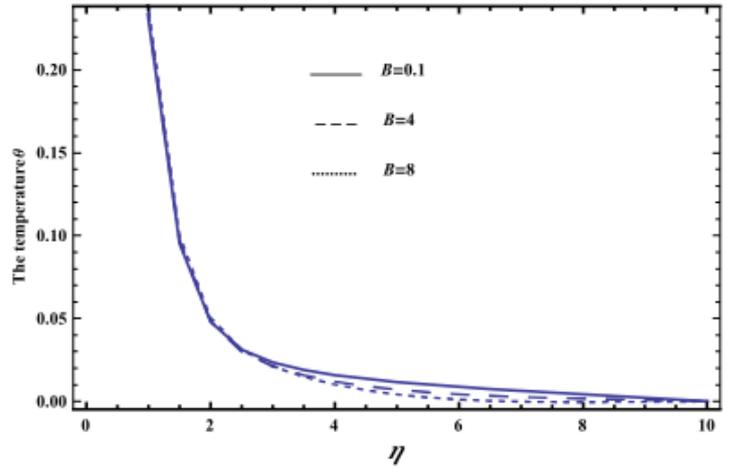
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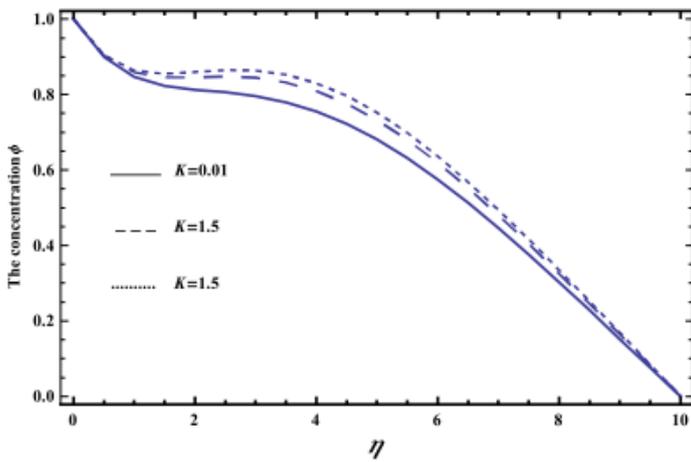
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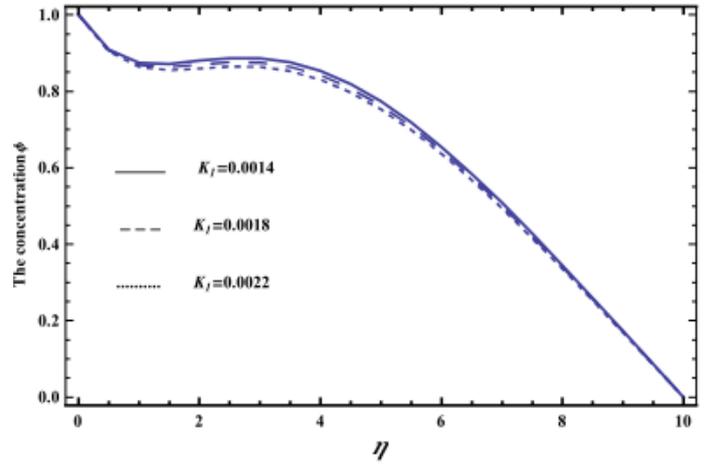
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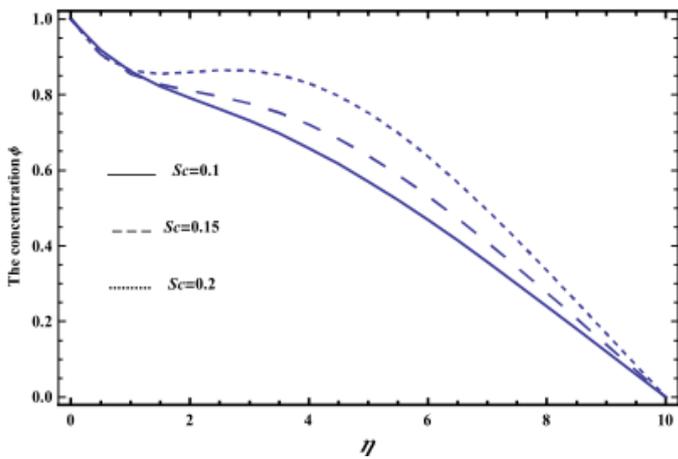
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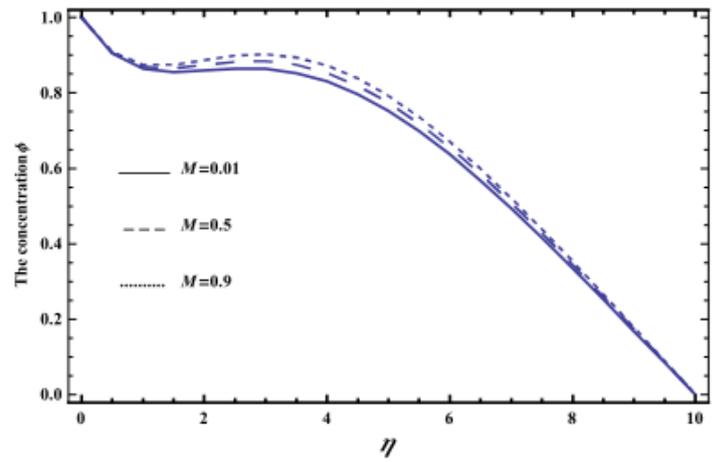
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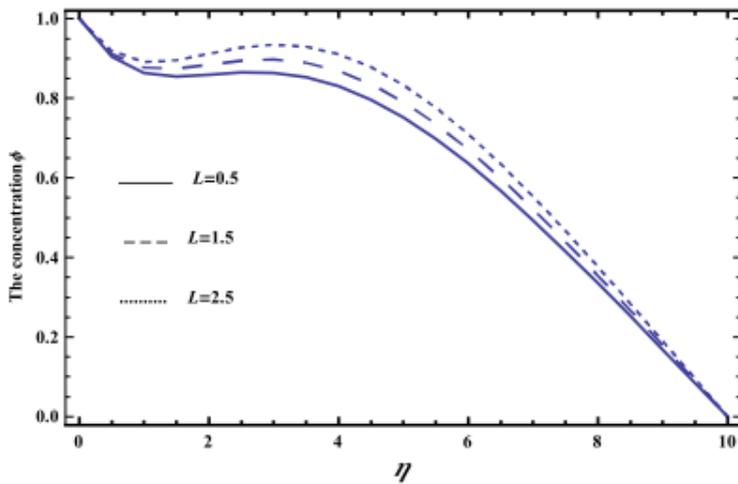
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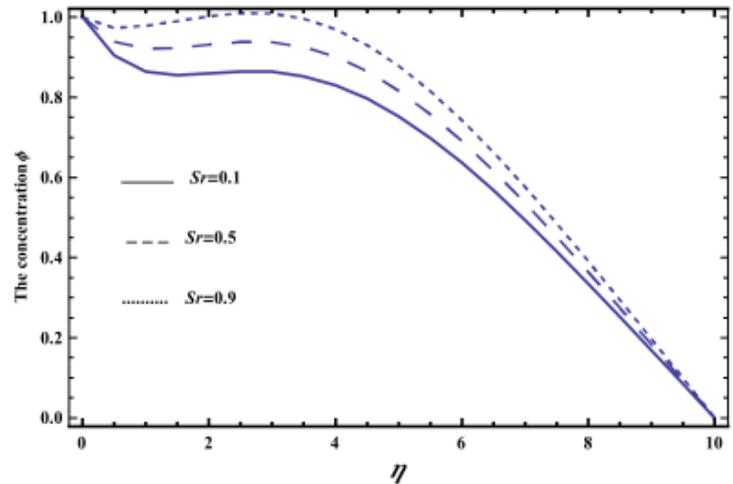
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FIGURE CAPTIONS

Fig. (1), the stream line ψ is plotted against η, for various values of the porous medium parameter $K = 0.01, 1.5, 2$, when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (2), the stream line ψ is plotted against η, for various values of the viscoelastic parameter $K_1 = 0.0014, 0.0018, 0.0022$, when $M = 0.01, K = 2$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (3), the stream line ψ is plotted against η, for various values of the magnetic field parameter $M = 0.01, 0.5, 0.9$, when $K = 2$, $K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (4), the temperature profile is plotted against η, for various values of the porous medium parameter $K = 0.01, 1.5, 2$, when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (5), the temperature profile is plotted against η, for various values of the viscoelastic parameter $K_1 = 0.0014, 0.0018, 0.0022$, when $M = 0.01, K = 2$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

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Fig. (7), the temperature profile is plotted against η, for various values of Eckert number $Ec = 0.001, 0.0012, 0.0014$, when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $K = 2$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (8), the temperature profile is plotted against η, for various values of the Prandtl number $Pr = 0.5, 0.7, 0.9$, when $M = 0.01, K_1 = 0.0022$, $K = 2$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$, $L = 0.5$.

Fig. (9), the temperature profile is plotted against η, for various values of the Defour number $Df = 0.2, 0.4, 0.6$, when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $K = 2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$

and $L = 0.5$.

Fig. (10), the temperature profile is plotted against η ,for various values of Schmidt number $Sc = 0.1, 0.15, 0.2$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $K = 2$ and $L = 0.5$.

Fig. (11), the temperature profile is plotted against η, for various values of the chemical reaction parameter $L = 0.5, 1.5, 2.5$, when $M = 0.01, K_1 = 0.0022$, $K = 2$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $Pr = 0.7$.

Fig. (12), the temperature profile is plotted against η ,for various values of Hartmann number $Ha = 1, 1.5, 2.5$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $K = 2$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (13), the temperature profile is plotted against η ,for various values of the Soret number $Sr = 0.1, 0.5, 0.9$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $K = 2$, $Ec = 0.001$, $Ha = 1.5$, $Sc = 0.2$ and $L = 0.5$.

Fig. (14), the temperature profile is plotted against η ,for various values of heat source sink parameter $\beta = 0.1, 4, 8$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $K = 2$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (15), the concentration profile is plotted against η ,for various values of the porous medium parameter $K = 0.01, 1.5, 2$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (16), the concentration profile is plotted against η, for various values of the viscoelastic parameter $K_1 = 0.0014, 0.0018, 0.0022$, when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (17), the concentration profile is plotted against η ,for various values of Schmidt number $Sc = 0.2, 1, 1.5$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (18), the concentration profile is plotted against η, for various values of the magnetic field parameter $M = 0.01, 0.5, 0.9$ when $K = 2, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $L = 0.5$.

Fig. (19), the concentration profile is plotted against η, for various values of the chemical reaction parameter $L = 0.5, 1.5, 2.5$, when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $Sr = 0.1$, $Sc = 0.2$ and $K = 2$.

Fig. (20), the concentration profile is plotted against η ,for various values of the Soret number $Sr = 0.1, 0.5, 0.9$,when $M = 0.01, K_1 = 0.0022$, $Pr = 0.7$, $\beta = 0.1$, $Df = 0.2$, $Ha = 1.5$, $Ec = 0.001$, $K = 2$, $Sc = 0.2$ and $L = 0.5$.